NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2599

EXPERIMENTAL DETERMINATION OF TIME CONSTANTS AND NUSSELT NUMBERS FOR BARE-WIRE THERMOCOUPLES IN HIGH-VELOCITY AIR STREAMS AND ANALYTIC APPROXIMATION OF CONDUCTION AND RADIATION ERRORS

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Page 20, equation (28): The numerical constant should be 0.0540 instead of 16.2.

Page 20, equation (29): The numerical constant should be 0.00171 instead of 0.246.

Page 48, equation (C7a): The exponent of T_w should be 3.815 instead of 3.82.

Page 48, equation (C7b): The numerical constant should be 0.93×10^{-10} instead of 0.97×10^{-10}

Page 49, equation (C8b): The numerical constant should be 4.05 instead of 4.20.

Page 50, equation (C11b): The numerical constant should be 1.71×10^{-3} instead of 1.65×10^{-3}

Page 69, figure 11(c): The ordinate values should be 10 times as large as the values indicated.

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SUMMARY

Experiments were conducted to determine time constants of several bare-wire thermocouples mounted in cross flow to an air stream. Four pairs of common thermocouple materials as well as three different wire diameters were used. The probes were tested at the exit of a room-temperature jet exhausting into a pressure controlled chamber. The Mach number range was 0.1 to 0.9; the range of Reynolds number based on wire diameter was 250 to 30,000.

For the configurations tested, thermocouple material, wire diameter, gas pressure, and Mach number were shown to influence the time constant. From these data, a correlation between Nusselt and Reynolds number was obtained for the range of Mach and Reynolds number stated. The correlation is

$$Nu = (0.427 \pm 0.018)(Re^*)^{(0.515 \pm 0.005)} Pr^{0.3}$$

or

$$Nu = (0.478 \pm 0.002)\sqrt{Re^*}\, Pr^{0.3}$$

with average deviations of a single observation of 6.9 and 7.4 percent, respectively. The value Re^* is a Reynolds number computed by evaluating gas viscosity and density at total temperature; Nu is Nusselt number computed by evaluating gas thermal conductivity at total temperature, and Pr is the Prandtl number of the gas.

Graphs and nomographs are presented for the computation of approximate radiation error, conduction error, and time constant for thermocouples.

Analytic determinations are also presented of the effects of dissimilarities in wire material and wire diameter on the steady-state temperature distribution in the exposed portion of the wire, and of the effect on time constant of conduction along the thermocouple wire to the supports.

INTRODUCTION

Performance evaluation and control of jet engines, as well as the fundamental study of the related combustion phenomena, depend considerably on a knowledge of the steady and variable temperatures of the gas within the engine. At present, these temperatures are most commonly measured with thermocouples. The measurement accuracy is generally dependent on the temperature level, the gas velocity, the temperature of the surrounding walls, and the thermocouple construction. The design of a thermocouple for any particular application represents a compromise among the contradictory factors of accuracy, life, ruggedness, and rapidity of response to changes as they are influenced by conduction and radiation losses, partial adiabatic recovery, erosion, size, and convective heat-transfer rate with the moving gas.

Whether a thermocouple is designed to respond rapidly to temperature changes or to measure accurately the steady-state temperature of the gas, the controlling factors are the same, although the orders of importance may be different. Although this report is concerned principally with speed of response, consideration of the related steady-state accuracy is included.

Limited data have been available for estimating time constants and these data are for certain thermocouple designs and for a small range of operating conditions. Data presented in reference 1 are time constants of bare-wire and radiation-shielded thermocouples at Reynolds numbers (based on wire diameter) from about 15 to 900 and Mach numbers from 0.05 to 0.14. The work was performed in exhaust gases at 1000° F, at gas velocities up to 250 feet per second, and with a step change of approximately 700° F. For a given thermocouple, the time constant was found to vary with the 0.5 power of the Reynolds number. Additional response-rate data on very fine wires for a Reynolds number range of 3 to 340 and a Mach number range of 0.02 to 0.1 are presented in reference 2. As shown in reference 2 and in other references, the study of response rates can be tantamount to the measurement of Nusselt number. The relation between Reynolds number and Nusselt number, and therefore response rate, for cylinders in cross flow is represented in a compilation of data in reference 3 for a Reynolds number range of 0.1 to 250,000 and Mach numbers up to 0.1. As gas velocity approaches the sonic value, it is conceivable that compressibility effects, as represented by the Mach number, may influence the Nusselt number. This influence is shown by the data presented in reference 4. These data were obtained in the Reynolds number range of 80 to 550 and Mach number range of 0.4 to 2.2.

The present work was undertaken to evaluate time constants of some representative types of thermocouple probe that are applicable for use in jet-engine operation. Experimental determinations were made of the time constants of bare-wire thermocouples in cross flow to an air stream for Reynolds numbers between 250 and 30,000 and for Mach numbers between 0.1 and 0.9. Nusselt numbers were computed from the experimental measurements. The ranges of Reynolds and Mach number reported by references 1 to 4, inclusive, as well as those presented herein, are shown in figure 1.

The associated theoretical analysis serves to evaluate the possible effects of conduction on the time constant and to provide an estimate of the conduction and radiation errors that can be expected in steady-state temperature indication. For the range of conditions covered, the data aid in establishing the relation among Nusselt, Reynolds, and Mach numbers to an accuracy adequate for estimating conduction and radiation errors and thermocouple time constants.

The work reported herein is part of a program of research in high-temperature measurements being conducted at the NACA Lewis laboratory.

THEORY

Method of theoretical treatment. - The theoretical treatment of this section will first define the concept of "time constant" as it appears in the most elementary case of convective heat transfer. Then, in the treatment of the more general case of combined convective, conductive, and radiative heat transfer from a single wire, the manner in which the time constant is affected by the presence of conduction and radiation will be shown. This treatment will lead to theoretical formulas for the approximate steady-state conduction and radiation errors. The interrelation between the thermocouple response rate and the factors affecting the steady-state errors will be demonstrated in the section Basic example. Next, there will be considered several more complex situations of practical interest: the single wire replaced by a thermocouple, a thermocouple with intermediate supporting wires, a single wire subjected to a step change from initial conditions different from those considered under Basic example, and a single wire heated electrically.

Finally, it will be shown how the introduction of empirically determined constants into the theoretical formulas may be used for the numerical computation of the approximate values of radiation error, time constant, and conduction error. The symbols used throughout this report and the detailed mathematical procedures that lead to the results presented in this section are given in appendixes A, and B and C, respectively.

The heat-transfer relation for a bare-wire thermocouple used to measure the temperature of an air stream will be developed under the following assumptions:

(a) The difference between thermocouple temperature and air-stream temperature is small compared with the absolute temperature level.

(b) The thermocouple dimensions are small compared with the dimensions of the duct or other solid enclosure.

(c) Radial temperature gradients in the wire may be neglected, so that the temperature at any cross section is constant throughout that cross section.

(d) The thermocouple is constructed in one of the forms shown in figure 2, with the thermocouple junction midway between the supports, and with the support temperature remaining constant or else changing very slowly as compared with the thermocouple-junction temperature.

Elementary case, excluding conduction and radiation. - In the absence of conductive and radiative heat transfer, the balance between rate of accumulation of heat in an element of length of wire and rate of convective heat transfer to the same element of length would be expressed by a simple total differential equation

$$\rho_w c_w V \frac{dT_w}{dt} = hA(T_g - T_w) \qquad (1a)$$

where

A surface area

c_w specific heat of wire

h convective heat-transfer coefficient

T_g effective gas temperature

T_w wire temperature

t time

V volume

ρ_w wire density

Equation (1a) may be written

$$\tau \frac{dT_w}{dt} + T_w = T_g \tag{1b}$$

where τ is the time constant of the wire. Characteristic of equation (1b) are the facts that

(a) In response to a sudden "step change" in gas temperature from an initial value $T_{g,1}$ (at which time T_w will be taken equal to $T_{g,1}$) to a new constant value $T_{g,2}$, the wire response will be given by

$$T_w = T_{g,2} + (T_{g,1} - T_{g,2})e^{-t/\tau} \tag{2}$$

(b) In response to a sinusoidal fluctuation in gas temperature, of angular frequency ω, represented by

$$T_g = T_{g,av} + \overline{T} \sin (\omega t)$$

and initiated at time $t = 0$ (at which time T_w will be taken as equal to $T_{g,av}$), the wire temperature will be given by

$$T_w = T_{g,av} + \frac{\overline{T}}{\sqrt{1+\omega^2 \tau^2}} \sin \left[\omega t - \tan^{-1} (\omega\tau)\right] + \frac{\overline{T}\, \omega\tau}{1+\omega^2 \tau^2} e^{-t/\tau} \tag{3}$$

General case. - In the presence of conductive and radiative heat transfer, equation (1) must be replaced by a partial differential equation involving x, distance along the wire, as a second independent variable. The solutions for response to a step change or to a sinusoidal fluctuation are correspondingly more complex. Examination of these equations and comparison with equations (2) and (3) indicate, however, that, under the assumptions initially stated, it is still possible to have a characteristic time constant τ for the thermocouple (although it may no longer be numerically equal to that appearing in equation (1b)) and the behavior of the thermocouple junction can still be described by equations of the form of equations (2) and (3).

In order to indicate the form of the time constant in the more general case, as well as to evaluate the errors in steady-state temperature indication that are caused by conduction and by radiation, the general heat-transfer equation will be derived.

For an element of length Δx of one of the wires, the rate of storage of heat q_p is equal to the sum of the rates of heat transfer into the wire element by the processes of conduction, convection, and radiation. These rates of heat transfer will be denoted by q_k, q_c, and q_r, respectively, so that

$$q_p \, \Delta x = (q_k + q_c + q_r) \, \Delta x \tag{4}$$

The following additional symbols will be used:

D wire diameter

k_g thermal conductivity of gas

k_w thermal conductivity of wire

L length of wire between supports

Nu Nusselt number

T_d equivalent duct temperature, defined as absolute temperature of a black-body enclosure (of included solid angle 4π) whose radiation to thermocouple, through gas, will be same as that of actual duct

$T_{w,m}$ absolute wire temperature at thermocouple junction

$\alpha_{g,d}$ effective absorptivity of gas for black-body radiation at temperature T_d

ϵ_g effective emissivity of gas

ϵ_w emissivity of wire

σ Stefan-Boltzmann constant

The rate of storage of heat in unit length of the wire is

$$q_p = \rho_w c_w \frac{\partial T_w}{\partial t} \frac{\pi D^2}{4} \tag{5}$$

The net rate of heat transfer by conduction through the ends of unit length of wire is

$$q_k = k_w \frac{\partial^2 T_w}{\partial x^2} \frac{\pi D^2}{4} \tag{6}$$

The rate of heat transfer by convection into the surface of the wire per unit length of wire is

$$q_c = \frac{Nu\ k_g}{D} (T_g - T_w)\ \pi D \tag{7}$$

From the surface of unit length of wire, the radiant power emitted is $\sigma\epsilon_w T_w^4 \pi D$, the power received from the gas is $\sigma\epsilon_w \epsilon_g T_g^4 \pi D$, and the power received from the duct by transmission through the gas is $\sigma\epsilon_w (1-\alpha_{g,d}) T_d^4 \pi D$. Hence, the net rate of heat transfer by radiation to the surface of unit length of wire is

$$q_r = \sigma\epsilon_w \left[(1-\alpha_{g,d})T_d^4 + \epsilon_g T_g^4 - T_w^4\right] \pi D \tag{8}$$

However, if $\left[(T_g - T_w)/T_g\right] << 1$, the approximation $T_g^4 - T_w^4 \approx 4T_g^3(T_g - T_w)$ may be introduced, permitting equation (8) to be written

$$q_r = \sigma\epsilon_w \left[(1-\alpha_{g,d})\ T_d^4 - (1-\epsilon_g)\ T_g^4\right] \pi D + 4T_g^3\ \sigma\epsilon_w\ (T_g - T_w)\ \pi D \tag{9}$$

An analysis of the shape factors that enter into the determination of T_d is presented in reference 5.

Substitution of equations (5), (6), (7), and (9) into equation (4) yields

$$\tau \frac{\partial T_w}{\partial t} = \frac{1}{\eta^2} \frac{\partial^2 T_w}{\partial x^2} + T_f - T_w \tag{10}$$

where τ, η, and T_f are defined by

$$\tau_1 = \frac{D^2 \rho_w c_w}{4 \ Nu \ k_g} \tag{11a}$$

$$\beta_1 = \frac{\sigma \ D \ T_g^{\ 4}}{Nu \ k_g} \tag{11b}$$

$$\eta_1^{\ 2} = \frac{4 \ Nu \ k_g}{D^2 \ k_w} \tag{11c}$$

$$\tau = \frac{\tau_1}{(1+4\beta_1 \epsilon_w/T_g)} \tag{11d}$$

$$\beta = \frac{\beta_1 \epsilon_w}{(1+4\beta_1 \epsilon_w/T_g)} \tag{11e}$$

$$\eta^2 = \eta_1^{\ 2} \ (1+4\beta_1 \epsilon_w/T_g) \tag{11f}$$

$$T_f = T_g + \beta \left[(1-\alpha_{g,d})\left(\frac{T_d}{T_g}\right)^4 - (1-\epsilon_g)\right] \tag{11g}$$

For purposes of future computation, it is also convenient to introduce the thermal diffusivity of the wire

$$\kappa = \frac{k_w}{\rho_w c_w} \tag{11h}$$

so that equation (11f) may also be written

$$\eta^2 = 1/\kappa\ \tau \tag{11i}$$

The physical significance of the quantities appearing in equations (11) becomes apparent upon examination of the solutions of equation (10) for some simple boundary and initial conditions, as treated in appendix B.

Under conditions of a steady gas temperature, the corresponding steady state of the wire $(\partial T_w/\partial t = 0)$ is one in which the balance of heat-transfer rates by conduction, convection, and radiation produces a wire temperature T_w that is different from the gas temperature T_g and is given by

$$T_w = T_g + \beta\left[(1-\alpha_{g,d})\left(\frac{T_d}{T_g}\right)^4 - (1-\epsilon_g)\right] + (T_b-T_f)\,\psi \tag{12a}$$

where $\psi = \psi(\eta,x)$, and T_b is the temperature at the supports, $x = 0$ and $x = L$.

The quantity T_f includes the effect of radiant heat transfer and represents the temperature the wire would assume if there were no conductive heat transfer. The "radiation error" is therefore given by

$$\beta\left[(1-\alpha_{g,d})\left(\frac{T_d}{T_g}\right)^4 - (1-\epsilon_g)\right] \tag{13a}$$

An alternative form for the radiation error is

$$\overline{\beta}\ \left[(1-\alpha_{g,d})\left(\frac{T_d}{T_w}\right)^4 - (1-\epsilon_g)\right] \tag{13b}$$

where

$$\overline{\beta} = \frac{\overline{\beta}_1\epsilon_w}{(1+4\overline{\beta}_1\epsilon_w\epsilon_g/T_w)} \tag{13c}$$

and

$$\overline{\beta}_1 = \frac{\sigma D T_w^4}{Nu\ k_g} \tag{13d}$$

This alternative form is useful when the indicated wire temperature T_w is known, and it is desired to determine the radiation correction to be applied to the reading in order to obtain T_g. The use of the alternative quantity $\overline{\beta}$ permits expression of equation (12a) in the form

$$T_g = T_w - \overline{\beta}\left[(1-\alpha_{g,d})\left(\frac{T_d}{T_w}\right)^4 - (1-\epsilon_g)\right] - \frac{(T_b-T_w)\,\psi}{1-\psi} \tag{12b}$$

The intermediate quantities β_1 and $\overline{\beta}_1$ are introduced merely for convenience in writing the equations. The effect of conductive heat transfer is to make the steady-state wire temperature different from T_f. The magnitude of this difference and hence the "conductive error" is a function of η and is directly proportional to (T_f-T_b). An additional effect of conductive heat transfer is to produce a nonuniform temperature distribution along the length of the wire, ranging from a value T_b at the supports to a value $T_{w,m}$ at the midpoint of the wire (thermocouple junction).

If both thermocouple wires have substantially the same diameter and thermal conductivity, they also have substantially the same η. The steady-state conduction error at the midpoint, where $x = L/2$, is then

$$(T_b-T_f)\,\psi_m \tag{14}$$

where, as shown in appendix B, section II,

$$\psi_m = \operatorname{sech}\,(\eta L/2) \tag{15}$$

If the gas temperature is changed to a new value, the wire approaches a new steady-state value, involving a new value of T_f. The approach to the new value is at a finite rate, determined by the quantities τ and η. In the absence of appreciable conductive and radiative heat transfer, the wire has a time constant τ_1; in the absence of conduction only, the time constant is τ; in the presence of a moderate amount of conduction, the thermocouple junction acts approximately as though it had a time constant

$$\tau\ (1-\psi_m) \tag{16}$$

Basic example. - An illustrative example is the simple case where both thermocouple wires have substantially the same diameter and thermal conductivity. If (a) the gas temperature is suddenly changed from a value $T_{g,1}$ to a new steady-state value $T_{g,2}$ (step change), (b) the thermocouple was initially in equilibrium with the gas, and (c) $T_{g,1} > T_{g,2} > T_b > T_d$, the response of the thermocouple junction is shown in figure 3(a), where the variation of $T_{w,m}$ with time is indicated. The initial and final distributions of temperature along the wire are shown in figure 3(b), where the variation of T_w with x/L is indicated at times $t = 0$ and $(t/\tau) \rightarrow \infty$. The derivation of the equation for steady-state temperature distribution along the wire is presented in section II of appendix B. The effect of parameter ηL upon the distribution is shown in figure 4. The equation for time response of the thermocouple junction is derived in section VI of appendix B, resulting in equations (B39) and (B40). If the wire is subjected to a sinusoidal variation of temperature rather than a step change, the time response of the thermocouple junction, derived in section VII of appendix B, is given approximately by equation (B53).

Comparison of equations (B40) and (B53) with equations (2) and (3), respectively, leads to the conclusion that the effect of conduction is to attenuate the temperature change by a factor of $(1-\psi_m)$, to multiply the time constant τ by the same factor, and to add a steady conduction error equal to ψ_m times the difference between the support temperature and the average impressed temperature T_f.

Other examples. - In addition to the simple example just discussed, four basic alternative situations are of interest. The first of these is the case when the two thermocouple wires have radically different values of the quantity $(k_w \eta D^2)$. The expressions for conduction error and effective time constant become more complex, but an equivalent quantity $(\eta' L)$ can be defined in terms of the quantities $(\eta_A L + \eta_B L)$, $(\eta_A L - \eta_B L)$, and $(k_w \eta D^2)_A/(k_w \eta D^2)_B$, where the subscripts A and B pertain to the two thermocouple wire materials. An equivalent quantity ψ'_m can be used in the expression for steady-state and transient response discussed previously. The steady-state conduction error for this situation is discussed in greater detail in section III of appendix B.

The second alternative situation of interest is one in which the thermocouple junction is made between two wires which are attached to the supports through intermediate wires of diameters different from those forming the junction (fig. 2(c)). Here again equivalent quantities $(\eta''L)$ and ψ''_m can be defined in terms of the ηL values and the $k_w \eta D^2$ ratios. The steady-state conduction error for the case where both thermocouple materials have substantially the same thermal conductivity is treated in detail in section IV of appendix B. Combination with the case discussed in the preceding paragraph is possible in order to include the effect of an appreciable difference between the thermal conductivities of the two wires.

The third alternative situation is one in which a step change to a constant value of gas temperature $T_{g,2}$ is applied to a thermocouple that is initially at support temperature T_b rather than being initially at equilibrium with the gas stream. This procedure is descriptive of the lag-testing technique described in reference 1. The thermocouple is initially shielded from the hot gases by a tube through which cooler air is blown. The tube is then suddenly removed, exposing the unit to the hot gases. The response is shown graphically in figure 5. Mathematically, it involves replacement of the multiplier $(T_{f,1}-T_{f,2})$ in equation (B40) by the multiplier $(T_b-T_{f,2})$.

The fourth alternative situation is represented by the experiments described in this report in which the heating effect of an electric current was used to raise the temperature of the wire artificially and the step change was produced when the current was broken. This situation involves the addition of a term $q_e \Delta x = W\,\Delta x$ to the right side of equation (4), where W is the power dissipated per unit length of wire. The effect of this term is merely to add the quantity $W/\pi\,Nu\,k_g$ to the right side of equation (11g) for T_f. The effective time constant is unaffected. The response is shown graphically in figure 6.

Computation of time constant, conduction error, and radiation error. - The Nusselt number Nu enters into formulas (11a) through (11f) for the constants τ_1, β_1, and η_1^2 that are required for the computation of radiation error, conduction error, and effective time constant. It is conventional in heat-transfer work to express this number in terms of the fluid properties and dynamic conditions by means of the relation

$$Nu = F(Re,\ Pr) \tag{17a}$$

where Re and Pr are the Reynolds number and Prandtl number, respectively. Because of the large range of aerodynamic conditions covered in these tests, the conditions being representative of the possible uses of the thermocouples, it was found appropriate to modify equation (17a) to include the Mach number M as a separate independent variable and to use a simple product of powers of the Reynolds and Prandtl numbers as part of the function F, thus yielding

$$Nu = \text{constant} \times Re^{a}\, Pr^{b}\, f(M) \tag{17b}$$

For the purpose of actual computation, it is desirable, so far as is convenient, to express the values of τ_1, β_1, and η_1^2 in terms of quantities that are actually measured. Such quantities are the total temperature and the static and total pressures. For greater convenience, the Mach number M is also introduced as an intermediate variable, its value in terms of static- to total-pressure ratio being readily available in tables. Then, utilization of the additional facts that (a) both the viscosity and the thermal conductivity of air can be expressed as proportional to a power of the temperature (appendix C) and (b) the Prandtl number for air probably enters as the 0.3 power and the value of $Pr^{0.3}$ may be treated as remaining constant (appendix C) shows that the Reynolds number and, consequently, the Nusselt number can be expressed as products of powers of the Mach number, static pressure, and static temperature. In appendix C, the expressions for Reynolds number and for thermal conductivity of air are shown to be

$$Re = \text{constant} \times DMpT_s^{-1.19}\sqrt{\gamma} \tag{18}$$

$$k_g = \text{constant} \times T^{0.78} \tag{19}$$

After these expressions are inserted into equation (17b) and the static temperature is used in the equation for thermal conductivity, equations (11a), (11b), and (11c) may be written

$$\tau_1 = \text{constant} \times \frac{\rho_w c_w D^{2-a}}{p^a M^a T_s^{0.78-1.19a}\, \gamma^{a/2}\, f(M)} \tag{20a}$$

$$\eta_1^{\,2} = \text{constant}\times\frac{p^a M^a T_s^{\,0.78-1.19a}\ \gamma^{a/2}\ f(M)}{k_w D^{2-a}} \tag{21a}$$

$$\beta_1 = \text{constant}\times\frac{\sigma\, D^{1-a} T_s^{\,3.22+1.19a}}{p^a M^a\ \gamma^{a/2}\ f(M)} \tag{22a}$$

where the Prandtl number has been included as a part of the constants.

If gas density, viscosity, and thermal conductivity are evaluated at total rather than at static temperature, an alternative Reynolds number Re^* is defined

$$Re^* = \text{constant}\times DMpT_t^{\,-1.19}\sqrt{\gamma\Big/\left(1 + \frac{\gamma-1}{2}M^2\right)}$$

(appendix C) and alternative forms of equations (17b), (20a), (21a), and (22a) are

$$Nu = \text{constant}\times(Re^*)^a\ Pr^b\ f^*(M) \tag{17c}$$

$$\tau_1 = \text{constant}\times\frac{\rho_w c_w D^{2-a}\left(1 + \frac{\gamma-1}{2}M^2\right)^{a/2}}{p^a M^a T_t^{\,0.78-1.19a}\ \gamma^{a/2}\ f^*(M)} \tag{20b}$$

$$\eta_1^{\,2} = \text{constant}\times\frac{p^a M^a T_t^{\,0.78-1.19a}\ \gamma^{a/2}\ f^*(M)}{k_w\ D^{2-a}\left(1 + \frac{\gamma-1}{2}M^2\right)^{a/2}} \tag{21b}$$

$$\beta_1 = \text{constant}\times\frac{\sigma\, D^{1-a} T_t^{\,3.22+1.19a}\left(1 + \frac{\gamma-1}{2}M^2\right)^{a/2}}{p^a M^a\ \gamma^{a/2}\ f^*(M)} \tag{22b}$$

APPARATUS AND PROCEDURE

High-velocity air source. - The tests were performed at the exit of a $2\frac{3}{4}$-inch-diameter jet that exhausted into a pressure controlled receiver (fig. 7). Performance characteristics of the jet are described in reference 6. At the test section, a Mach number of 0.1 to 0.9 and a free-stream density of 0.04 to 0.10 pound mass per cubic foot were available. By individual control of the jet inlet and receiver pressures, any combination of Mach number and density for the ranges listed was obtainable. Each run was made at constant test-section density and varying Mach number. The total temperature of the air was between 540° and 545° R and was constant to within 1° during any one reading of a run. The total pressure and total temperature in the plenum chamber, and the static pressure at the test section, with the probe inserted into the air stream, were used to calculate Mach number and free-stream density.

Test probes. - Six thermocouple probes were tested; their dimensions and configurations are shown in figure 8. The four pairs of wire materials used were chromel-constantan, chromel-alumel, iron-constantan, and platinum plus 13-percent rhodium - platinum. Probes A, B, and C were of the same wire material but of increasing wire diameter; probes B, D, E, and F were of approximately the same size and configuration but of different wire materials. A seventh thermocouple was also tested for purposes of comparison with probe B. This thermocouple, identified by the letter G, was a straight chromel-constantan bare wire stretched across the exhaust of the jet.

Time-constant determinations. - The time-constant determinations were made without moving the probe or shielding it from the air stream. In order to simulate a higher initial gas temperature, an electric current was passed through the thermocouple wire to heat the junction to about 100° R above room temperature. A double-pole, double-throw snap action toggle switch was then used to disconnect the thermocouple from the source of heating current and to connect it to a recording oscillograph. As indicated under THEORY, this technique served to simulate a step change from one gas temperature to another. It also served to minimize any change in the nature of the temperature distribution along the wire. This distribution itself was, for all wires tested, such that the contribution of the term $\tau\psi_m$ to the effective time constant was negligible. For this reason, and because of the technique used for measuring the time constant as described later, it was also possible to neglect without serious error two deviations from the assumptions of the simple theory. These deviations were: first, the base of the thermocouple loop, constituting the "supports," actually did change temperature moderately; and second, for the iron-constantan wires an appreciable difference exists between the electric power dissipation in the two halves of the thermocouple loop and, consequently, an asymmetry in temperature distribution and a difference in the values of T_f occur. For example, in the most serious case, that of probe D at $M = 0.1$ and $\rho_g = 0.04$ pound mass per cubic foot, the value of ηL is 6.6 for the iron wire and 11.0 for the constantan wire, corresponding to a correction of 3 percent if the support temperature is assumed equal to the initial wire temperature. The ratio of ηL values for the two halves of the wire was 1.7:1 and the ratio of increase in T_f values due to the heating current was 5:1.

As the thermocouple was cooled, its output was recorded by a critically damped moving-coil oscillograph element having a natural period of 0.006 second and hence an effective time constant of 0.002 second. The record was obtained upon photographic paper that was marked off in 0.01-second intervals. The paper speed and oscillograph-element sensitivity were constant for all practical purposes. A typical record is shown in figure 9(a). Figure 9(b), a graph of the same data on semilogarithmic paper, shows that in the region following the "Start of record" the relation is sufficiently linear that a single measurement, directly on the oscillograph record, of the time required for the recorded temperature to have covered 63.2 percent of the temperature interval from start to asymptote is sufficient to establish the time constant for each of the runs. The time constants were read to a probable error of 0.005 second. For the temperature range encountered in these experiments, it was reasonable to assume that the wire calibration (emf/deg temperature change) remained constant. Each of the measured values of time constant was used as obtained from the records since the maximum conduction error by equation (16) was never more than 3 percent and usually about 1/2 to 1 percent.

RESULTS AND DISCUSSION

Time constants and Nusselt numbers. - The data obtained in the experiments are shown in figure 10 as a plot of the experimentally measured time constants against the Reynolds number Re based on static pressure and temperature. The conduction correction factor ψ_m is negligible for almost all the data, being no more than 3 percent in the extreme case. The radiation correction is also negligible. Consequently, the experimentally measured time constants are assumed to be the same as the quantity τ_1 of equation (11a).

It will be noted that points in the lower Mach number region fall on lines having a constant slope of approximately 0.5 and that points in the higher Mach number region show systematic deviations from these lines of constant slope. This effect is more strongly indicated by plots of time constant as a function of Mach number, as shown in figure 11. On the average, these figures indicate that the deviations begin at approximately $M = 0.5$.

An alternative presentation of the data is a plot of the time constant against the Reynolds number Re^* based on an evaluation of gas density and viscosity at total temperature rather than at static temperature. Such plots (fig. 12) show appreciably smaller deviations at higher Mach numbers than were in evidence in figure 10 and are equivalent to assuming for the functions $f(M)$ and $f^*(M)$ in equations (17b) and (17c) the forms

$$f(M) = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1.69a} \tag{23a}$$

$$f^*(M) = 1 \tag{23b}$$

These expressions follow from the fact that

$$Re^* = Re \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1.69} \tag{24}$$

(See appendix C.) If Re^* replaces Re, it is reasonable to assume that the total temperature, rather than the static temperature, should be used in evaluating the thermal conductivity. Making this assumption and utilizing the values of mean $\rho_W c_W$ from tables I or II and of thermal conductivity of air at the operating total temperature permit computation of the Nusselt number by equation (11a). A plot of this computed Nusselt number against the Reynolds number Re^* is shown in figure 13 for all the 176 data points taken in these tests. The least-squares solution for the best straight line through these points, assuming that all Nusselt number measurements have equal probable percentage error, is

$$Nu = (0.385 \pm 0.016)(Re^*)^{(0.515 \pm 0.005)} \tag{25a}$$

in the Reynolds number range $250 < Re^* < 30,000$. The average deviation of any point from the line is 6.9 percent. If the exponent of Re^* is fixed at the value $\frac{1}{2}$, the least-squares solution for the multiplying constant leads to

$$Nu = (0.431 \pm 0.002)\sqrt{Re^*} \tag{26a}$$

with an average deviation of any point of 7.4 percent. Formula (26a) will be used in future discussion because of its greater convenience and negligibly greater average deviation.

If the Nusselt number - Reynolds number relation is to be applied to gases other than air, the Prandtl number Pr may be inserted explicitly in equations (25a) and (26a) by writing them

$$Nu = (0.427\pm0.018)(Re^*)^{(0.515\pm0.005)}\ Pr^{0.3} \tag{25b}$$

$$Nu = (0.478\pm0.002)\sqrt{Re^*}\ \ Pr^{0.3} \tag{26b}$$

where the exponent b in equation (17b) has been set equal to 0.3 in accordance with the conclusions reported in reference 3, chapter VIII. Formulas (25a) and (26a) are independent of any assumptions regarding the value of Prandtl number; formulas (25b) and (26b) are based on a value of $Pr = 0.71$ for air at 540° R.

Some of the sources of error, in estimated order of importance, are lack of roundness and smoothness of the wire and junction, error in effective wire-diameter measurement, deviation from simple theory because of changing support temperature, inaccuracy in measurement of time constant, uncertainty in knowledge of $\rho_w c_w$, use of the arithmetically averaged $\rho_w c_w$ for the two thermocouple materials, and omission of the conduction correction. The accuracy of the correlation represented by equations (25) and (26) is dependent on the accuracy of knowledge of gas viscosity and thermal conductivity at the operating total temperature (540° R); because all tests were made at constant total temperature, the accuracy of the correlation and therefore formulas (25) and (26), are independent of assumptions made concerning the variation of Prandtl number, gas viscosity, and thermal conductivity with temperature.

A few experiments performed with probe C at a gas temperature of 1500° R showed that the formulas presented remain applicable. The data are not presented herein.

The compilation of data presented in chapter VIII of reference 3 gives, for the range $M < 0.1$, $1000 < Re < 50{,}000$,

$$Nu = 0.26(Re)^{0.60}$$

Data (at higher gas temperatures) from reference 1 give the value of 0.5 for the exponent of Reynolds number for the range $M < 0.14$, $20 < Re < 1000$. Data from reference 4, at $M = 0.38$, give

$$Nu = 0.47 \sqrt{Re}$$

The data in references 3 and 4 were based on an evaluation of gas properties at a temperature substantially equal to the arithmetic average of wire temperature and total gas temperature. For purposes of comparison, the lines represented by formulas (25a) and (26a) are shown in figure 13 together with the curve given in reference 3.

Effective gas temperature T_g. - Most satisfactory correlation between time constant and Reynolds number is achieved by evaluating gas density and viscosity at total temperature. This technique can be rationalized by considering that the wire temperature T_w substantially represents the average temperature of the gas immediately surrounding the wire and that this layer of gas offers the principal resistance to the flow of heat. A graph of recovery temperature ratio T_w/T_t of a cylinder in cross flow as a function of Mach number is shown in figure 14. The data are the composite results of a large number of experiments performed at this laboratory during the past 5 years in connection with other research projects. These measurements were made so that conduction and radiation errors were minimized and therefore T_g would be approximately equal to T_w for these data. These data agree quite well with values reported from time to time in the scientific literature. The cross-hatched section on the graph indicates the average spread of all the available data, which include the effect of different free-stream air densities. The ratio of T_s/T_t is also shown. It is evident that the recovery temperature T_w is much closer to the total temperature T_t than to the static temperature T_s. Reynolds numbers computed at $M = 0.9$ based on gas properties evaluated at T_t, T_w, and T_s would be in the ratio of 1.00:1.08:1.29, respectively.

The finding that a better correlation of heat-transfer data is often achieved when gas properties are evaluated at surface temperature was first reported in reference 7 for low-velocity flow and has recently been verified at higher velocities in reference 8. The experimenters in references 7 and 8 were concerned with flow in pipes, where viscosity is the predominant factor, rather than with flow around cylinders, where potential flow effects are as important as viscous effects. Consequently, the physical explanation of the finding that the use of total temperature provided a better correlation than the use of static temperature in the case of flow around cylinders is not necessarily of the same nature as the explanation of the heat-transfer phenomena in pipes.

NUMERICAL COMPUTATION OF TIME CONSTANT,

RADIATION ERROR, AND CONDUCTION ERROR

The Nusselt number, once having been determined from experimental measurements, may then be inserted into the basic theoretical formulas for time constant, conduction error, and radiation error. Since, as shown in appendix C, air viscosity and thermal conductivity may be expressed as functions of absolute temperature, it follows that Reynolds number, Nusselt number, radiation error, time constant, and conduction error, may also be expressed in terms of static pressure, total pressure, and total temperature, all of which are experimentally measurable quantities. The computation procedure is facilitated by introducing Mach number, the value of which is readily obtained from tables if the static and total pressures are known.

The insertion of equation (26a) into equations (20b), (21b), and (22b) leads to the equations

$$\tau_1 = \frac{4.05\ \rho_w c_w D^{1.50}\left(1 + \frac{\gamma-1}{2} M^2\right)^{0.25}}{p^{0.50}\ M^{0.50}\ T_t^{0.18}} \quad \text{seconds} \tag{27}$$

$$\beta_1 = \frac{16.2\ \sigma\ D^{0.50}\ T_t^{3.82}\left(1 + \frac{\gamma-1}{2} M^2\right)^{0.25}}{p^{0.50}\ M^{0.50}}\ {}^{\circ}\mathrm{R} \tag{28}$$

$$\eta_1^2 = \frac{0.246\ p^{0.50}\ M^{0.50}\ T_t^{0.18}}{k_w\ D^{1.50}\left(1 + \frac{\gamma-1}{2} M^2\right)^{0.25}} \quad \text{inches}^{-2} \tag{29}$$

where the quantities are in the following units:

$\rho_w c_w$ Btu/(ft^3 $^{\circ}$R)

D (in.)

p (atm)

T ($^{\circ}$R)

k_w Btu/(ft ^{o}R sec)

σ 0.173×10^{-8} Btu/(hr ft^2 $^{o}R^4$)

These formulas show that time constant and conduction error are only slightly affected by the magnitude of the gas temperature; hence the validity of these formulas is not strongly dependent on any assumptions made concerning dependence upon temperature of the gas viscosity and thermal conductivity. The time constant computed from equation (27) has been plotted in figure 15 against the experimentally measured time constants. As is to be expected from the fact that figure 15 is substantially an alternative presentation of the data shown in figure 13, the average deviation of the points shown in figure 15 is also 6.9 percent. The detailed derivations of numerical formulas such as equations (27), (28), and (29) are presented in appendix C and lead to the construction of graphs and nomograms (figs. 16 to 19) that materially facilitate the computations of time constant, radiation error, and conduction error for wire diameters between 0.001 and 0.1 inch. The exact computation procedure follows:

Given:

(1) Wire-material constants: ϵ_w, ρ_w, c_w, k_w or κ

(2) Probe constants: D, L, L', T_b, T_w

(3) Constant of surroundings: T_d

(4) Gas and aerodynamic constants: M, p, $\alpha_{g,d}$, ϵ_g

Procedure:

(1) Find $\overline{\beta}_1$ from figure 16(a).

(2) Compute $\overline{\beta} = \overline{\beta}_1 \epsilon_w/(1+4\overline{\beta}_1\epsilon_w\epsilon_g/T_w) \approx \overline{\beta}_1\epsilon_w$

(3) Compute radiation error = $\overline{\beta}\left[(1-\alpha_{g,d})(T_d/T_w)^4-(1-\epsilon_g)\right]$ using the nomograph of figure 16(b).

(4) Find τ_{Pt} from figure 17.

(5) Compute $\tau_1 = \tau_{Pt}\,(\rho_{w,A}\,c_{w,A} + \rho_{w,B}\,c_{w,B})/2\rho_{Pt}c_{Pt}$ using data in figure 17.

(6) Compute $\tau = \tau_1/(1+4\overline{\beta}_1\epsilon_w\epsilon_g/T_w)$

This value is the time constant in the absence of conduction effects.

(7a) Compute the effective $(\eta L)^2 = L^2/\tau\kappa$ using the harmonically averaged value of κ from table II.

(7b) Alternatively, find $(\eta_1 L)_{Pt}^2$ from figure 18 and compute, for each thermocouple wire,

$$(\eta_1 L)_w^2 = (\eta_1 L)_{Pt}^2 \; (k_{Pt}/k_w)$$

and

$$(\eta L)_w^2 = (\eta_1 L)_w^2 \; (1+4\bar{\beta}_1 \epsilon_w \epsilon_g / T_w)$$

where (k_{Pt}/k_w) is listed in table I. Then find the effective (ηL) for the thermocouple wire pair by use of the equations

$$m_A/m_B = (k_{w,A}/k_{w,B})^{0.5} \; (D_A/D_B)^{1.25}$$

$$m_{A,B} = \left[(m_A/m_B)-1\right] / \left[(m_A/m_B)+1\right]$$

and of tables II and III.

(8) Find ψ_m from figure 19.

(9) Compute steady-state conduction error = $(T_b - T_w)\,\psi_m$

(10) Compute corrected time constant = $\tau(1 - \psi_m)$

(11) Compute corrected gas temperature $T_g = T_w$ - (steady-state radiation error) - (steady-state conduction error).

SUMMARY OF RESULTS

The time constants of bare-wire thermocouples mounted in cross flow to an air stream were shown to depend principally on thermocouple wire material, wire diameter, gas pressure, and Mach number, and only slightly on gas temperature. These measurements also served to provide a correlation among Mach number M, Nusselt number Nu, and Reynolds number Re^* for the ranges $0.1 \leq M \leq 0.9$ and $250 \leq Re^* \leq 30,000$, yielding

$$Nu = (0.385 \pm 0.016)(Re^*)^{(0.515 \pm 0.005)}$$

with a 6.9 percent average deviation of a single observation, or

$$\overline{Nu} = (0.431 \pm 0.002)\sqrt{Re^*}$$

with a 7.4 percent average deviation of a single observation. The value Re^* is a Reynolds number computed by evaluating gas viscosity and density at total temperature. The value of Nu was computed from the experimental measurement of time constant and the evaluation of gas thermal conductivity at total temperature. Prandtl number was considered constant for these tests and is therefore incorporated in the constants. The Prandtl number Pr may be inserted explicitly in the preceding equations by writing them

$$Nu = (0.427 \pm 0.018)(Re^*)^{(0.515 \pm 0.005)}\, Pr^{0.3}$$

$$Nu = (0.478 \pm 0.002)\sqrt{Re^*}\, Pr^{0.3}$$

It is shown that the evaluation of Nusselt number provides a means for determining approximate steady-state radiation and conduction errors of a bare-wire thermocouple in a high-temperature gas stream. Graphs and nomograms are presented for the computation of time constant, approximate radiation error, and approximate conduction error for wire diameters between 0.001 and 0.1 inch for commonly used pairs of thermocouple materials.

Analytic determinations are presented of the effects of dissimilarities in wire material or wire diameter on steady-state temperature distribution in a thermocouple, and the effect on the time constant of conduction along the thermocouple wire to the support.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, August 3, 1951

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	surface area
c	specific heat
D	wire diameter
h	heat-transfer coefficient
k	thermal conductivity
L	length of wire between supports
L'	length of central wire when intermediate supporting wires are used
M	free-stream Mach number
m	$k D^2 \eta$
$m_{A,B}$	$\left[(m_A/m_B)-1\right]/\left[(m_A/m_B)+1\right]$
Nu	Nusselt number
Pr	Prandtl number
p	static pressure
q_c	rate of heat transfer by convection, per unit length
q_e	rate of heat transfer by electric power dissipation, per unit length
q_k	rate of heat transfer by conduction, per unit length
q_p	rate of heat storage, per unit length
q_r	rate of heat transfer by radiation, per unit length
Re	Reynolds number with gas properties computed at static temperature

Re^*	Reynolds number with gas properties computed at total temperature
r	specific gas constant for air, 1716 $ft^2/(sec^2)(^{o}R)$
T	temperature
$\overline{T}$	amplitude of sinusoidal gas-temperature fluctuation
T_d	equivalent duct temperature
T_f	final wire temperature in absence of conduction
t	time
U	linear free-stream velocity
u	intermediate temperature variable
V	volume
v	intermediate temperature variable
W	electrical power per unit length
w	intermediate temperature variable
x	distance along wire
Y,y,z	intermediate temperature variables
$\alpha_{g,d}$	effective absorptivity of gas for black-body radiation at temperature T_d
β	$\beta_1\epsilon_w/(1+4\beta_1\epsilon_w/T_g)$
$\overline{\beta}$	$\overline{\beta}_1\epsilon_w/(1+4\overline{\beta}_1\epsilon_w\epsilon_g/T_w)$
β_1	$\sigma\ DT_g^{\ 4}/Nu\ k_g$
$\overline{\beta}_1$	$\sigma\ DT_w^{\ 4}/Nu\ k_g$
γ	ratio of specific heats
γ_0	ratio of specific heats of air at NACA standard sea-level conditions, 1.40
ϵ_g	effective emissivity of gas

ϵ_w	emissivity of wire
η	$\eta_1 \ (1+4\bar{\beta}_1 \epsilon_w \epsilon_g / T_w)^{1/2}$
η_1	$(4 \ \mathrm{Nu} \ k_g / D^2 \ k_w)^{1/2}$
θ	intermediate time variable
κ	thermal diffusivity of wire
μ	gas viscosity
ξ	intermediate distance variable
ρ	density
σ	Stefan-Boltzmann constant
τ	time constant, $\tau_1/(1+4\bar{\beta}_1 \epsilon_w \epsilon_g / T_w)$
τ_1	$D^2 \ \rho_w \ c_w / 4 \ \mathrm{Nu} \ k_g$
τ_{Pt}	value of τ_1 for platinum wire
τ_0	value of τ_{Pt} at NACA standard sea-level conditions
Φ	transient temperature function
ψ	spatial temperature distribution function
ω	angular frequency

Subscripts:

A	refers to wire material A
av	arithmetic average
B	refers to wire material B
b	value at wire support
g	refers to gas
i	value at $x = (L-L')/2$ and at $x = (L+L')/2$
m	value at $x = L/2$

P	refers to wire diameter D_P
Pt	value for platinum
Q	refers to wire diameter D_Q
s	refers to static conditions
t	refers to total conditions
w	refers to wire
0	refers to NACA standard sea-level conditions
1	initial value
2	final value

APPENDIX B

MATHEMATICAL SOLUTIONS OF HEAT-TRANSFER EQUATION FOR SOME BASIC DESIGNS

I. General Equation

Subject to the approximations and assumptions listed under THEORY, the general equation (10) is derived from a consideration of the balance between the rate of heat transfer to an element of length of the wire and the rate of storage of heat in the element. The general equation is repeated here for convenience:

$$\tau \frac{\partial T_w}{\partial t} = \frac{1}{\eta^2} \frac{\partial^2 T_w}{\partial x^2} + T_f - T_w \tag{B1}$$

For consideration of the transient response, it will be convenient to transform equation (B1) into

$$\tau \frac{\partial y}{\partial t} = \frac{1}{\eta^2} \frac{\partial^2 y}{\partial x^2} - y \tag{B2}$$

by a shift of the origin of ordinates,

$$y \equiv T_w - T_f \tag{B3}$$

so that y represents the difference between instantaneous wire temperature and the temperature which the wire would ultimately assume in the absence of conduction.

In general, T_w and y are functions of x and of t. The values of these two temperatures at the midpoint of the wire, at which the thermocouple junction is located, will be denoted by $T_{w,m}$ and y_m, respectively. The characteristic parameters τ and η, plus the initial and boundary conditions, then serve to define fully the values of $T_w(x,t)$, $y(x,t)$, $T_{w,m}(t)$, and $y_m(t)$. The conduction parameter η governs the steady-state temperature distribution; the time constant τ exercises the principal influence on the transient behavior, although the transient behavior is also influenced by η.

II. Steady-State Temperature Distribution in Single Wire Stretched between Supports

If both thermocouple materials have essentially the same diameter and thermal conductivity, the physical situation shown in figure 2(a) may be treated as though a single wire, possessing characteristic parameters τ and η, were stretched between supports at constant temperature T_b. In the presence of a constant gas temperature T_g, the steady state of temperature distribution in the wire is then represented by setting $\partial T_w/\partial t$ equal to zero in equation (B1), so that the equation becomes

$$\frac{1}{\eta^2} \frac{d^2 T_w}{dx^2} + T_f - T_w = 0 \tag{B4a}$$

subject to the boundary conditions

$$T_w = T_b \quad \text{at} \quad x = 0 \tag{B4b}$$

$$T_w = T_b \quad \text{at} \quad x = L \tag{B4c}$$

The solution of this equation is

$$T_w = T_f + (T_b - T_f)\psi \tag{B5}$$

where

$$\psi = \frac{\sinh \eta x + \sinh (\eta L - \eta x)}{\sinh \eta L} \tag{B6}$$

In particular, the value of ψ at the midpoint of the wire $x = L/2$ will be denoted by ψ_m, so that

$$T_{w,m} = T_f + (T_b - T_f)\psi_m \tag{B7}$$

where

$$\psi_m = \operatorname{sech}\ (\eta L/2) \tag{B8}$$

Figure 4 shows graphs of ψ against x/L for various values of the dimensionless parameter ηL. It is to be noted that only in the decade $1 < \eta L < 10$ is ψ_m markedly different from the values 0 or 1.

III. Steady-State Temperature Distribution along Thermocouple Wire Pair Stretched between Supports

In the general case where a thermocouple composed of wires having different diameters or thermal conductivities is stretched between supports at constant temperature T_b, subscripts A and B will be assigned to represent the two wire materials. The situation is illustrated by figure 2(b). To treat this problem, as well as the one in the following section, it is convenient to utilize the following:

Lemma: The solution of the equation (B4a) for steady-state temperature distribution along a single wire subject to the boundary conditions

$$T_w = T_\alpha \quad \text{at} \quad x = x_\alpha \tag{B9a}$$

$$T_w = T_\beta \quad \text{at} \quad x = x_\beta \tag{B9b}$$

is

$$T_w = T_f + \frac{(T_\alpha - T_f)\ \sinh\ (\eta x - \eta x_\beta) + (T_\beta - T_f)\ \sinh\ (\eta x_\alpha - \eta x)}{\sinh\ (\eta x_\alpha - \eta x_\beta)} \tag{B10}$$

For the problem of two thermocouple wires, A and B, the differential equations and their associated boundary conditions are:

For wire A

$$0 \leq x \leq \frac{L}{2}$$

$$\left.\begin{aligned} \frac{1}{\eta_A^2}\frac{d^2 T_{w,A}}{dx^2} + T_{f,A} - T_{w,A} &= 0 \\ T_{w,A} = T_b \quad &\text{at} \quad x = 0 \\ T_{w,A} = T_{w,m} \quad &\text{at} \quad x = \frac{L}{2} \end{aligned}\right\} \tag{B11}$$

For wire B

$$\frac{L}{2} \leq x \leq L$$

$$\left.\begin{aligned} \frac{1}{\eta_B^2}\frac{d^2 T_{w,B}}{dx^2} + T_{f,B} - T_{w,B} &= 0 \\ T_{w,B} = T_{w,m} \quad &\text{at} \quad x = \frac{L}{2} \\ T_{w,B} = T_b \quad &\text{at} \quad x = L \end{aligned}\right\} \tag{B12}$$

It will be noted that continuity of T_w at $x = L/2$ has already been postulated by the preceding expressions, and that the unknown $T_{w,m}$ has been used as one of the boundary values. In order to determine $T_{w,m}$, the additional postulate is made that the net rate of flow of heat across the thermocouple junction shall be zero. Since the rate of flow of heat along wire A into the junction is

$$k_{w,A} \frac{\pi D_A^2}{4} \frac{d T_{w,A}}{dx}$$

and the rate of flow of heat along wire B from the junction is

$$k_{w,B} \frac{\pi D_B^2}{4} \frac{d T_{w,B}}{dx}$$

it is necessary that

$$k_{w,A}\, D_A^{\,2}\, \frac{d\, T_{w,A}}{dx} = k_{w,B}\, D_B^{\,2}\, \frac{d\, T_{w,B}}{dx} \quad \text{at} \quad x = \frac{L}{2} \tag{B13}$$

Equations (B11), (B12), and (B13) constitute the complete statement of the problem.

By use of the lemma previously stated, appropriate values for T_α and T_β are inserted into equation (B10) to obtain solutions of equations (B11) and (B12). Inserting the value $x = L/2$ into these solutions and then applying the condition stated by equation (B13) give the solution for $T_{w,m}$ as

$$T_{w,m}\left[m_A \coth\left(\frac{\eta_A L}{2}\right) + m_B \coth\left(\frac{\eta_B L}{2}\right)\right] = T_{w,A}\, m_A \coth\left(\frac{\eta_A L}{2}\right) + T_{f,B}\, m_B \coth\left(\frac{\eta_B L}{2}\right) + (T_b - T_{f,A})\, m_A \operatorname{csch}\left(\frac{\eta_A L}{2}\right) + (T_b - T_{f,B})\, m_B \operatorname{csch}\left(\frac{\eta_B L}{2}\right) \tag{B14}$$

where

$$\left.\begin{aligned} m_A &= k_{w,A}\, D_A^2\, \eta_A \\ m_B &= k_{w,B}\, D_B^{\,2}\, \eta_B \end{aligned}\right\} \tag{B15}$$

Expression (B14) may be simplified if the steady-state radiation errors for the two wire materials are assumed to be sufficiently similar that $T_{f,A}$ and $T_{f,B}$ may each be replaced by a common value T_f (for example, $T_{f,av} = (T_{f,A} + T_{f,B})/2$). Under this assumption, equation (B14) becomes

$$T_{w,m} = T_{f,av} + (T_b - T_{f,av})\psi'_m \tag{B16}$$

where

$$\psi'_m = \operatorname{sech}\left(\frac{\eta_A L+\eta_B L}{4}\right) \cosh\left(\frac{\eta_A L-\eta_B L}{4}\right) \frac{1-m_{A,B} \tanh\left(\frac{\eta_A L-\eta_B L}{4}\right) \coth\left(\frac{\eta_A L+\eta_B L}{4}\right)}{1-m_{A,B} \sinh\left(\frac{\eta_A L-\eta_B L}{2}\right) \operatorname{csch}\left(\frac{\eta_A L+\eta_B L}{2}\right)} \tag{B17}$$

and

$$m_{A,B} = \frac{\frac{m_A}{m_B} - 1}{\frac{m_A}{m_B} + 1} \tag{B17b}$$

Thus an equivalent value ψ'_m may be defined to replace the quantity ψ_m in equation (B7). Alternatively, an equivalent quantity $\eta'L$ may be defined to replace the quantity ηL in equation (B8), where η' will lie between η_A and η_B. A tabulation of $\eta'L$ as a function of the dimensionless quantities $\eta_A L$, $\eta_B L$, and $m_{A,B}$ is given in table III.

The experiments described in this report show that, for the range of Reynolds and Mach number covered in these experiments, the Nusselt number varies directly as the square root of wire diameter at any given set of gas-flow conditions. Hence, if radiation is neglected so that η may be replaced by η_1 in equation (11f), the application of equations (11c) and (B15) leads to

$$\frac{m_A}{m_B} = \left(\frac{k_{w,A}}{k_{w,B}}\right)^{0.5} \left(\frac{D_A}{D_B}\right)^{1.25} \tag{B18}$$

Values of $(k_{w,A}/k_{w,B})^{0.5}$ are given in table II for standard thermocouple wire pairs to facilitate computation of $m_{A,B}$.

It is evident that if both wires have substantially the same diameter and thermal conductivity, both $(\eta_A - \eta_B)$ and $m_{A,B}$ are close to zero, and ψ'_m given by equation (B17a) is substantially the same as ψ_m given by equation (B8).

Typical temperature distribution patterns along the wire for various values of $m_{A,B}$, in the case where $\eta_A L = 4$ and $\eta_B L = 10$, are shown in figure 20. It is evident that, depending on the value of $m_{A,B}$, the thermocouple junction temperature will lie somewhere between the values of $T_{w,m}$ for each of the two wires as given by figure 4 or equations (B7) and (B8).

IV. Steady-State Temperature Distribution along Wire Stretched between Intermediate Wire Supports of Different Diameter

If, as shown in figure 2(c), the wires constituting the thermocouple junction are supported by intermediate wires of different diameter, the intermediate wires also exposed to the gas stream but with their bases at fixed support temperature T_b, the values of η will be different for the intermediate wires even though they are of the same material as the adjacent wires forming the junction.

For simplicity, it will be assumed that both intermediate wires have the same diameter, both wires forming the junction have the same diameter, and both material A and material B have the same thermal conductivity. (The results of the previous section can serve to correct for deviations from these assumptions.) Under these assumptions, the intermediate wires, each of length $(L - L')/2$, may be assigned an η value of η_P and the two wires forming the junction may be treated as a single wire of length L' with an η value of η_Q, the subscripts P and Q now serving to distinguish between the two diameters. By virtue of symmetry, the temperature $T_{w,m}$ of the midpoint of wire Q is closer to the effective gas temperature $T_{f,Q}$ than is the temperature of any other point of the assembly. Similarly, for reasons of symmetry, the temperature of each junction between the central wire and the intermediate wires may be represented by the common symbol $T_{w,i}$.

A line of reasoning similar to that of section III regarding the rate of heat flow across the junction between central and intermediate wires leads to the following mathematical statement of the problem:

$$\left.\begin{aligned} &\frac{1}{{\eta_P}^2}\frac{d^2T_{w,P}}{dx^2} + T_{f,P} - T_{w,P} = 0 \\ &T_{w,P} = T_b \quad \text{at} \quad x = 0 \\ &T_{w,P} = T_{w,i} \quad \text{at} \quad x = \frac{L-L'}{2} \end{aligned}\right\} \quad 0 \leq x \leq \frac{L-L'}{2} \tag{B19}$$

$$\left.\begin{aligned} &\frac{1}{{\eta_Q}^2}\frac{d^2T_{w,Q}}{dx^2} + T_{f,Q} - T_{w,Q} = 0 \\ &T_{w,Q} = T_{w,i} \quad \text{at} \quad x = \frac{L-L'}{2} \\ &T_{w,Q} = T_{w,i} \quad \text{at} \quad x = \frac{L+L'}{2} \end{aligned}\right\} \quad \frac{L-L'}{2} \leq x \leq \frac{L+L'}{2} \tag{B20}$$

$$k_{w,P} D_P^2 \frac{d\,T_{w,P}}{dx} = k_{w,Q} D_Q^2 \frac{d\,T_{w,Q}}{dx} \quad \text{at} \quad x = \frac{L-L'}{2} \tag{B21}$$

The presence of symmetry makes it unnecessary to state or to use the relations for $x > (L+L')/2$. They would be similar to equations (B19) and (B21).

Use of the lemma stated in section III, insertion of the boundary conditions from equations (B19) and (B20) into equation (B10), and use of equation (B21) to eliminate the intermediate quantity $T_{w,i}$ yield the following solution for $T_{w,m}$:

$$(T_{w,m} - T_{f,Q})\left[1 + \frac{m_Q}{m_P} \tanh\left(\frac{\eta_Q L'}{2}\right) \tanh\left(\frac{\eta_P L - \eta_Q L'}{2}\right)\right] =$$

$$\operatorname{sech}\left(\frac{\eta_Q L'}{2}\right)\left[(T_b - T_{f,P}) \operatorname{sech}\left(\frac{\eta_P L - \eta_Q L'}{2}\right) + T_{f,P} - T_{f,Q}\right] \tag{B22}$$

where

$$\left.\begin{aligned} m_P &= k_{w,P} D_P^2 \eta_P \\ m_Q &= k_{w,Q} D_Q^2 \eta_Q \end{aligned}\right\} \tag{B23}$$

Equation (B22) may be simplified if it is assumed that the effects of the steady-state radiation errors are sufficiently alike that $T_{f,P}$ and $T_{f,Q}$ on the right side of the equation may each be replaced by a common value T_f (for example, $T_{f,av} = (T_{f,P} + T_{f,Q})/2$). Under this assumption, equation (B22) becomes

$$T_{w,m} = T_{f,av} + (T_b - T_{f,av})\,\psi''_m \tag{B24}$$

where

$$\psi''_m = \frac{\operatorname{sech}\left(\frac{\eta_Q L'}{2}\right) \operatorname{sech}\left[\frac{\eta_Q\,(L-L')}{2}\right]}{1 + \frac{m_Q}{m_P} \tanh\left(\frac{\eta_Q L'}{2}\right) \tanh\left[\frac{\eta_P\,(L-L')}{2}\right]} \tag{B25}$$

Thus equivalent values ψ''_m and $\eta''L$ may be so defined as to replace the quantities ψ_m and ηL in equations (B7) and (B8). A tabulation of $\eta''L$ as a function of the dimensionless quantities $\eta_P(L-L')$, $\eta_Q L'$, and (m_Q/m_P) is given in table IV. Because both materials have been assumed of the same thermal conductivity, equation (B18) and the discussion in section III pertaining to this equation show that m_Q/m_P is given very simply by

$$\frac{m_Q}{m_P} = \left(\frac{D_Q}{D_P}\right)^{1.25} \tag{B26}$$

By similar reasoning, it can also be shown that

$$\frac{\eta_Q}{\eta_P} = \left(\frac{D_P}{D_Q}\right)^{0.75} \tag{B27}$$

Temperature distribution patterns for a few cases of practical interest are shown in figure 21.

V. Transient and Steady-State Response of Single Wire to Step Change in Gas Temperature, Wire Initially at Support Temperature

This case, because of its relative mathematical simplicity, provides a convenient introduction to the subject of transient response of the wire. The general equation (B2) and its corresponding boundary and initial conditions for this case are

$$\left.\begin{aligned}
&\tau\,\frac{\partial y}{\partial t} = \frac{1}{\eta^2}\frac{\partial^2 y}{\partial x^2} - y \qquad y \equiv T_w - T_f \\
&y = T_b - T_f \quad \text{at} \quad x = 0 \\
&y = T_b - T_f \quad \text{at} \quad x = L \\
&y = T_b - T_f \quad \text{at} \quad t = 0
\end{aligned}\right\} \tag{B28}$$

Assume a solution of the form

$$y = u + v \tag{B29a}$$

where $u = u(x)$, with

$$\left.\begin{aligned} &\frac{1}{\eta^2}\frac{d^2u}{dx^2} - u = 0 \\ &u = T_b - T_f \quad \text{at} \quad x = 0 \\ &u = T_b - T_f \quad \text{at} \quad x = L \end{aligned}\right\} \tag{B29b}$$

and $v = v(x,t)$, with

$$\left.\begin{aligned} &\tau\frac{\partial v}{\partial t} = \frac{1}{\eta^2}\frac{\partial^2 v}{\partial x^2} - v \\ &v = 0 \quad \text{at} \quad x = 0 \\ &v = 0 \quad \text{at} \quad x = L \\ &v = T_b - T_f - u(x) \quad \text{at} \quad t = 0 \end{aligned}\right\} \tag{B29c}$$

The solution of equation (B29b) has already been derived in section II. This solution provides the steady-state solution for y, which is rewritten here as

$$u = (T_b - T_f)\psi \tag{B30}$$

where ψ is defined by equation (B6).

The solution of equation (B29c) will provide the transient response. Solution of the equation is facilitated by the substitution

$$w = w(x,t) = ve^{-t/\tau} \tag{B31}$$

which converts equation (B29c) into

$$\left.\begin{aligned} \eta^2 \tau \frac{\partial w}{\partial t} &= \frac{\partial^2 w}{\partial x^2} \\ w = 0 \quad &\text{at} \quad x = 0 \\ w = 0 \quad &\text{at} \quad x = L \\ w = T_b - T_f - u(x) = (T_b - T_f)(1 - \psi) \quad &\text{at} \quad t = 0 \end{aligned}\right\} \tag{B32}$$

The solution of equation (B32) is (reference 9, paragraph 34)

$$w = \sum_{n=1}^{\infty} \left[\frac{2}{L}\int_0^L (T_b-T_f)(1-\psi) \sin\left(\frac{n\pi\xi}{L}\right) d\xi\right] \sin\left(\frac{n\pi x}{L}\right) e^{-\frac{n^2\pi^2 t}{\eta^2 L^2 \tau}} \tag{B33}$$

Carrying through the integration and then returning to the variable v yield the solution of equation (B29c) as

$$v = (T_b - T_f)\Phi \tag{B34a}$$

where

$$\Phi = \frac{2}{\pi} e^{-t/\tau} \sum_{n=1}^{\infty} \frac{1-\cos(n\pi)}{n} \frac{1}{1+n^2\pi^2/\eta^2 L^2} \sin\left(\frac{n\pi x}{L}\right) e^{-\frac{n^2\pi^2 t}{\eta^2 L^2 \tau}} \tag{B34b}$$

The term $\left[1 - \cos (n\pi)\right]$ makes the series alternating, with only odd values of n present.

Addition of the expressions for u and v to obtain y (equation (B29a)) and replacement of y by $(T_w - T_f)$ yield the general solution

$$T_w = T_f + (T_b - T_f)\psi + (T_b - T_f)\Phi \tag{B35}$$

where ψ and Φ are given by equations (B6) and (B34b), respectively. At the midpoint of the wire, $x = L/2$ and equation (B35) becomes

$$T_{w,m} = T_f + (T_b - T_f)\psi_m + (T_b - T_f)\Phi_m \tag{B36a}$$

where

$$\psi_m = \operatorname{sech}(\eta L/2) \tag{B8}$$

$$\Phi_m = \frac{4}{\pi} e^{-t/\tau} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right) \frac{1}{1+n^2\pi^2/\eta^2L^2} e^{-\frac{n^2\pi^2 t}{\eta^2 L^2 \tau}} \tag{B36b}$$

At $t = 0$, Φ_m is equal to $(1 - \psi_m)$. For $t \geq 0$, Φ_m may be represented very closely (to within 0.03) by the approximation

$$\Phi_m \approx (1-\psi_m)\, e^{-t/\left[\tau(1-\psi_m)\right]} \tag{B36c}$$

so that an adequate approximation for $T_{w,m}$ is

$$T_{w,m} = T_f + (T_b - T_f)\psi_m + (T_b - T_f)(1 - \psi_m)e^{-t/\left[\tau(1 - \psi_m)\right]} \tag{B36d}$$

The physical significance of this solution has been discussed briefly in the section THEORY and illustrated in figure 5. The asymptote of the temperature-time curve is $T_f - (T_f - T_b)\psi_m$. The initial slope is $(T_f - T_b)/\tau$, which is independent of the conduction error factor ψ_m. The effective time constant is $\tau(1 - \psi_m)$. Thus, when there is appreciable conduction, the indicated magnitude of ultimate temperature change is reduced by an amount $(T_f - T_b)\psi_m$ from its "true" value T_f, but the time constant is altered in the same proportion in such manner as to maintain the initial rate of change equal to that which would be characteristic of the zero-conduction case.

VI. Transient and Steady-State Response of Single Wire to Step Change in Gas Temperature, Wire Initially at Gas Temperature

If the wire is initially at a temperature $\left[T_{f,1} + (T_b - T_{f,1})\psi\right]$ corresponding to an actual gas temperature $T_{g,1}$ related to $T_{f,1}$ by an equation of the form of (11g), and a step change in gas temperature occurs to a value of $T_{g,2}$ yielding a corresponding effective gas temperature $T_{f,2}$ and, by section II, an ultimate wire temperature $\left[T_{f,2} + (T_b - T_{f,2})\psi\right]$, then the mathematical expression of the problem calls for solving equation (B2) subject to the boundary and initial conditions

$$\left.\begin{aligned} y &= T_b - T_{f,2} \quad \text{at} \quad x = 0 \\ y &= T_b - T_{f,2} \quad \text{at} \quad x = L \\ y &= T_{f,1} + (T_b - T_{f,1})\psi - T_{f,2} \quad \text{at} \quad t = 0 \end{aligned}\right\} \tag{B37}$$

Assuming, as in section V, a solution of the form

$$y = u + v \tag{B38a}$$

where

$$u = u(x) \qquad u = T_b - T_{f,2} \quad \text{at} \quad x = 0 \quad \text{and at} \quad x = L \tag{B38b}$$

$$\left.\begin{aligned} v &= v(x,t) \qquad v = 0 \quad \text{at} \quad x = 0 \quad \text{and at} \quad x = L \\ v &= T_{f,1} - T_{f,2} + (T_b - T_{f,1})\psi - u \quad \text{at} \quad t = 0 \end{aligned}\right\} \tag{B38c}$$

a procedure similar to that of section V yields the solution

$$T_w = T_{f,2} + (T_b - T_{f,2})\psi + (T_{f,1} - T_{f,2})\Phi \tag{B39}$$

which at $x = L/2$ becomes

$$T_{w,m} = T_{f,2} + (T_b - T_{f,2})\psi_m + (T_{f,1} - T_{f,2})\Phi_m \tag{B40}$$

where ψ, Φ, ψ_m, and Φ_m are defined by equations (B6), (B34b), (B8), and (B36b), respectively.

In accordance with the approximation (B36c), the physical significance of the solution is illustrated by figure 3(a). The initial value of the indicated temperature-time curve is $\left[T_{f,1} + (T_b - T_{f,1})\psi_m\right]$; the asymptote is $\left[T_{f,2} + (T_b - T_{f,2})\psi_m\right]$; the indicated magnitude of ultimate change is $(T_{f,1} - T_{f,2})(1 - \psi_m)$; the initial slope is $(T_{f,1} - T_{f,2})/\tau$ and is independent of the conduction error factor ψ_m; the effective time constant is $\tau(1 - \psi_m)$. Thus, when there is appreciable conduction, the indicated magnitude of ultimate temperature change is reduced by a factor $(1 - \psi_m)$, but the apparent time constant is reduced in the same proportion, so that the initial rate of change remains the same as in the zero-conduction case.

VII. Transient and Steady-State Response of Single Wire to Suddenly Applied Sinusoidal Variation in Gas Temperature

If the wire, initially at temperature $\left[T_{f,av} + (T_b - T_{f,av})\psi\right]$, is subjected, at time $t = 0$, to a sinusoidal variation $\overline{T}\sin(\omega t)$ in gas temperature superimposed on the steady-state value $T_{f,av}$, the differential equation with its boundary and initial conditions is

$$\left.\begin{aligned}
\tau\frac{\partial T_w}{\partial t} &= \frac{1}{\eta^2}\frac{\partial^2 T_w}{\partial x^2} + T_{f,av} + \overline{T}\sin(\omega t) - T_w \\
T_w &= T_b \quad \text{at} \quad x = 0 \\
T_w &= T_b \quad \text{at} \quad x = L \\
T_w &= T_{f,av} + (T_b - T_{f,av})\psi \quad \text{at} \quad t = 0
\end{aligned}\right\} \tag{B41}$$

Introduction of the transformations

$$Y = T_w - T_{f,av} - \overline{T} \sin (\omega t) \tag{B42}$$

$$z = Y e^{t/\tau} - T_{f,av}\left(\frac{\omega\tau}{1+\omega^2\tau^2}\right) e^{t/\tau}\left[\cos (\omega t) + \omega\tau \sin (\omega t)\right] \tag{B43}$$

changes equation (B41) to

$$\left.\begin{aligned}
&\tau \frac{\partial z}{\partial t} = \frac{1}{\eta^2} \frac{\partial^2 z}{\partial x^2} \\
&z=\left[T_b - T_{f,av} - \overline{T} \sin(\omega t)\right] e^{t/\tau} + \overline{T} \frac{\omega\tau}{1+\omega^2\tau^2} e^{t/\tau}\left[\cos(\omega t) + \omega\tau \sin (\omega t)\right] \\
&\qquad \text{at} \quad x = 0 \quad \text{and at} \quad x = L \\
&z = (T_b - T_{f,av})\psi + \frac{\overline{T}\omega\tau}{1+\omega^2\tau^2} \quad \text{at} \quad t = 0
\end{aligned}\right\} \tag{B44}$$

The solution of equation (B44) is (reference 9, section 34)

$$\begin{aligned}
z = \frac{2}{L}\sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 t}{\eta^2 L^2 \tau}} \sin\left(\frac{n\pi x}{L}\right)&\left\{\int_0^L \left[(T_b - T_{f,av})\psi + \overline{T} \frac{\omega\tau}{1+\omega^2\tau^2}\right] \sin\left(\frac{n\pi\xi}{L}\right) d\xi \; + \right. \\
&\frac{n\pi}{\eta^2 L\tau}\int_0^t e^{\left(1 + \frac{n^2\pi^2}{\eta^2 L^2}\right)\frac{\theta}{\tau}}\left[1 - \cos (n\pi)\right]\left[T_b - T_{f,av} - \overline{T} \sin (\omega\theta) \; + \right. \\
&\left.\left.\overline{T} \frac{\omega\tau}{1+\omega^2\tau^2} (\cos \omega\theta + \omega\tau \sin \omega\theta)\right] d\theta\right\}
\end{aligned} \tag{B45}$$

For convenience, write

$$a_n \equiv \frac{2}{n\pi}\left[1 - \cos (n\pi)\right] \sin \frac{n\pi x}{L} \tag{B46}$$

$$b_n \equiv 1 + n^2 \pi^2/\eta^2 L^2 \tag{B47}$$

Then, performance of the integrations in equation (B45) and return to the variable T_w by use of equations (B42) and (B43) result in

$$T_w = T_{f,av} + (T_b - T_{f,av})\psi +$$

$$\overline{T}\frac{\sqrt{A^2+\omega^2\tau^2B^2}}{\sqrt{1+\omega^2\tau^2}} \sin\left[\omega t - \tan^{-1}(\omega\tau) + \tan^{-1}\left(\frac{\omega\tau B}{A}\right)\right] + \overline{T}\omega\tau C \tag{B48a}$$

where

$$A = 1 - \sum_{n=1}^{\infty} a_n \frac{b_n(b_n-1)}{b_n^2+\omega^2\tau^2} \tag{B48b}$$

$$B = \sum_{n=1}^{\infty} a_n \frac{b_n - 1}{b_n^2+\omega^2\tau^2} \tag{B48c}$$

$$C = \sum_{n=1}^{\infty} a_n \frac{e^{-b_n t/\tau}}{b_n^2+\omega^2\tau^2} \tag{B48d}$$

and use has been made of the identities

$$1 \equiv \sum_{n=1}^{\infty} a_n \tag{B49}$$

$$\psi \equiv \sum_{n=1}^{\infty} a_n \frac{b_n-1}{b_n} \tag{B50}$$

At the midpoint of the wire, where $x = L/2$, equation (B48) becomes

$$T_{w,m} = T_{f,av} + (T_b - T_{f,av})\psi_m +$$

$$\overline{T}\,\frac{\sqrt{A_m^2+\omega^2\tau^2B_m^2}}{\sqrt{1+\omega^2\tau^2}}\sin\left[\omega\tau - \tan^{-1}(\omega\tau)+\tan^{-1}\left(\frac{\omega\tau B_m}{A_m}\right)\right]+\overline{T}\,\omega\tau\, C_m \qquad \text{(B51a)}$$

where

$$A_m = 1 - \sum_{n=1}^{\infty} a_{n,m}\frac{b_n(b_n-1)}{b_n^2+\omega^2\tau^2} \qquad \text{(B51b)}$$

$$B_m = \sum_{n=1}^{\infty} a_{n,m}\frac{b_n-1}{b_n^2+\omega^2\tau^2} \qquad \text{(B51c)}$$

$$C_m = \sum_{n=1}^{\infty} a_{n,m}\frac{e^{-b_n t/\tau}}{b_n^2+\omega^2\tau^2} \qquad \text{(B51d)}$$

$$\psi_m = \operatorname{sech}\,(\eta L/2) \qquad \text{(B8)}$$

$$a_{n,m} \equiv \frac{4}{n\pi}\sin\frac{n\pi}{2}$$

Approximations (to about 0.03 in amplitude and to about 3° in angle) for the functions appearing in equation (B51) are

$$\frac{\sqrt{A_m^2 + \omega^2\tau^2B_m^2}}{\sqrt{1 + \omega^2\tau^2}} \approx \frac{(1 - \psi_m)}{\sqrt{1 + \omega^2\tau^2(1 - \psi_m)^2}}$$

$$\tan^{-1}\omega\tau - \tan^{-1}\left(\frac{\omega\tau B_m}{A_m}\right) \approx \tan^{-1}\left[\omega\tau(1 - \psi_m)\right]$$

$$C_m \approx (1 - \psi_m)^2 \, e^{-t/\left[\tau(1-\psi_m)\right]} \Big/ \left[1 + \omega^2\tau^2 \, (1 - \psi_m)^2\right]$$

so that an approximate solution is

$$T_{w,m} = T_{f,av} + (T_b - T_{f,av})\psi_m + \overline{T} \, \frac{1-\psi_m}{\sqrt{1+\omega^2\tau^2(1-\psi_m)^2}} \sin\left\{\omega t - \tan^{-1}\left[\omega\tau \, (1-\psi_m)\right]\right\} +$$

$$\overline{T}\,\omega\tau \frac{(1-\psi_m)^2}{1+\omega^2\tau^2(1-\psi_m)^2} \, e^{-t/\left[\tau(1-\psi_m)\right]} \qquad \text{(B53)}$$

Thus, by comparison with equation (3), the effect of conduction is to make the system act as though it possessed a time constant $\tau(1 - \psi_m)$ rather than τ. At the same time, the amplitude of the steady-state term and the amplitude of the transient term are both attenuated by a factor $(1 - \psi_m)$. These conclusions are similar to those obtained for the case of a step change in effective gas temperature.

APPENDIX C

NUMERICAL COMPUTATIONS

The time constant and the steady-state radiation and conduction error may be expressed, to an accuracy sufficient for all practical purposes, in terms of the experimentally measured static pressure and total temperature and of the Mach number, the value of which is readily obtainable from the experimentally measured static and total pressures. The numerical formulas for the intermediate quantities required for the numerical expression of the approximate values of radiation error, time constant, and conduction error will be derived in sequence.

Viscosity. - The viscosity μ of air is independent of pressure and can be expressed in terms of the temperature by means of the formulas

$$\mu = \mu_0 \left(\frac{T}{T_0}\right)^{0.69} \tag{C1a}$$

$$\mu = 0.159\times10^{-6}\ T^{0.69}\ \text{lbm}/\left[(\text{sec})(\text{ft})\right] \tag{C1b}$$

where T is in $^{\circ}$R, to within 1 percent in the range $460^{\circ} < T < 2300^{\circ}$ R, if values given by reference 3, quoting a personal communication from Genereaux, are assumed. In equation (C1a), μ_0 is the viscosity of air at temperature T_0. Equation (C1b) is based on assuming μ_0 equal to 11.9×10^{-6} pound mass per second per foot at $T_0 = 519^{\circ}$ R. It will be assumed that formula (C1b) can be extrapolated to the interval $400^{\circ} < T < 3000^{\circ}$ R.

Reynolds number. - The Reynolds number Re can be expressed in terms of Mach number, pressure, temperature, and wire diameter by use of formula (C1) for the viscosity, the universal gas law, and the formula for the velocity of sound

$$\text{Re} = \frac{UD\rho g}{\mu} = \frac{1}{\mu_0}\left(\frac{T_s}{T_0}\right)^{-0.69} M\sqrt{\gamma r T_s}\ \ D\,\frac{p}{rT_s} \tag{C2a}$$

where r is the specific gas constant for air. Formula (C2a) may be written

$$\text{Re} = 1.01\times10^{9}\ DMpT_s^{-1.19}\ \sqrt{\gamma/\gamma_0} \tag{C2b}$$

where D is in inches, p is in atmospheres, and T is in $^{\circ}$R. The Reynolds number so defined involves the evaluation of the gas properties at static pressure and static temperature. If, in order to improve the correlation between heat transfer and aerodynamic conditions, a different Reynolds number Re* is defined that involves evaluation of the gas density and viscosity at total rather than at static temperature, the conversion formula

$$T_t = T_s \left(1 + \frac{\gamma-1}{2} M^2\right) \tag{C3}$$

may be applied to equation (C2) to yield

$$Re^* = \frac{1}{\mu_0}\left(\frac{T_0}{T_t}\right)^{0.69} M\sqrt{\gamma r T_s}\, D\, \frac{p}{rT_t} = Re\left(\frac{T_s}{T_t}\right)^{1.69} \tag{C4a}$$

This formula may be written

$$Re^* = 1.01\times10^9\, D\, M\, p\, T_t^{-1.19}\left[\frac{\gamma_0}{\gamma}\left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{-1/2} \tag{C4b}$$

if D, p, and T are in inches, standard atmospheres, and $^{\circ}$R, respectively, and γ_0 (= 1.40) is the value of γ at NACA standard sea-level conditions.

<u>Thermal conductivity</u>. - Because adequate experimental data on the thermal conductivity of air at high temperatures are unavailable, the thermal conductivity k_g will be computed on the assumptions that the Prandtl number is constant and equal to 0.71, the viscosity is given by equation (C1), the specific heat of air is given by the values tabulated in reference 10 for the temperature range $460^{\circ} < T < 4000^{\circ}$ R, and the specific heat - temperature relation can be extrapolated to 400° R. Since little information is available regarding the variation of Prandtl number of air with temperature, and some of the information is contradictory, it has been recommended by Dr. E. R. G. Eckert of the Lewis laboratory that an assumption of constant Prandtl number be made. The value suggested by Dr. Eckert is the latest value at room temperature, as given by reference 11. The formulas

$$k_g = k_{g,0}\, (T/T_0)^{0.78} \tag{C5a}$$

$$k_g = 3.03\times10^{-8}\, T^{0.78}\ \mathrm{Btu}/[(\mathrm{ft})(\mathrm{sec})(^{\circ}\mathrm{R})] \tag{C5b}$$

where T is in $^{\circ}R$, then fit the computed values within 2 percent in the range $400^{\circ} < T < 3000^{\circ}$ R. In formula (C5a), $k_{g,0}$ is the thermal conductivity of air at temperature T_0. In formula (C5b), the value of $k_{g,0}$ is taken as 4.0×10^{-6} Btu/[(ft)(sec)($^{\circ}$R)] at $T_0 = 519^{\circ}$ R.

Nusselt number. - Insertion of formulas (C4a) and (C4b) into equation (26a) leads to

$$Nu = 0.431 \left[\frac{1}{\mu_0} \left(\frac{T_0}{T_t} \right)^{0.69} D\ M\ p \sqrt{\frac{\gamma}{rT_t \left(1 + \frac{\gamma - 1}{2} M^2 \right)}} \right]^{1/2} \tag{C6a}$$

$$Nu = 1.37 \times 10^4 \ (D\ M\ p)^{1/2} \ T_t^{-0.60} \left[\frac{\gamma_0}{\gamma} \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{-1/4} \tag{C6b}$$

where, in equation (C6b), the units of D, p, and T_t are inches, atmospheres, and $^{\circ}$R, respectively.

The computations of the quantities β_1, τ_1, and η_1 require knowledge of the thermal conductivity of air. The uncertainty in knowledge of this quantity is sufficiently large that no appreciable increase in total error results by use of $\gamma = 1.34$, as an average value over the temperature range $500^{\circ} < T_t < 3000^{\circ}$ R, for computation of β_1, τ_1, and η_1. Formula (C6b) then becomes

$$Nu = 1.36 \times 10^4 \ (D\ M\ p)^{1/2} \ T_t^{-0.60} \ (1 + 0.2M^2)^{-1/4} \tag{C6c}$$

Steady-state radiation error. - Computation of the steady-state radiation error is conveniently accomplished by first evaluating the quantity $\overline{\beta}_1$. Insertion of equation (C5) (evaluated at total temperature) and equation (C6) into equation (13d) and replacement of the quantity $T_t^{0.18}$ by the almost numerically equal quantity $T_w^{0.18}$ yield

$$\overline{\beta}_1 = \frac{2.32\ \sigma\ \mu_0^{1/2} r^{1/4} T_0^{0.435}}{k_{g,0}} \left(\frac{1 + \frac{\gamma - 1}{2} M^2}{\gamma} \right)^{1/4} \left(\frac{D}{Mp} \right)^{1/2} T_w^{3.82} \tag{C7a}$$

$$\overline{\beta}_1 = 0.97 \times 10^{-10} T_w^{3.82} \sqrt{\frac{D}{Mp}} \ (1 + 0.2M^2)^{1/4} \quad (^{\circ}R) \tag{C7b}$$

where, in equation (C7b), the units, D, p, and T_w are inches, atmospheres, and °R, respectively. The nomograph of figure 16(a) yields $\bar{\beta}_1$ when M, D, p, and T_w are given.

The radiation error may then be computed as

$$\text{Radiation error} = \bar{\beta}\left[(1 - \alpha_{g,d})\left(\frac{T_d}{T_w}\right)^4 - (1 - \epsilon_g)\right] \qquad (13b)$$

where

$$\bar{\beta} = \frac{\bar{\beta}_1 \epsilon_w}{1 + 4\bar{\beta}_1 \epsilon_w \epsilon_g / T_w} \approx \bar{\beta}_1 \epsilon_w \qquad (13c)$$

In the absence of conduction, the actual gas temperature T_g is then obtained by subtracting the radiation error from the indicated wire temperature T_w.

The quantity $\bar{\beta}_1 \epsilon_w$ represents approximately the error present when $T_d/T_w = 1.2$. As an aid to computation, a nomograph for $(T_d/T_w)^4$ is presented in figure 16(b).

Time constant. - The computation of τ_1, the time constant in the absence of radiation or conduction, is accomplished by inserting equations (C5) and (C6) into equation (11a), yielding

$$\tau_1 = \frac{0.58\mu_0^{1/2} r^{1/4} T_0^{0.435}}{k_{g,0}} \frac{\rho_w c_w D^{3/2}}{(M\ p)^{1/2} T_t^{0.18} \gamma^{1/4}} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/4} \qquad (C8a)$$

$$\tau_1 = 4.20 \frac{\rho_w c_w D^{3/2}}{(M\ p)^{1/2} T_t^{0.18}} (1 + 0.2M^2)^{1/4} \text{ sec} \qquad (C8b)$$

where, in equation (C8b), the units are

$\rho_w c_w$ Btu/[(ft^3)(°R)]

D inches

p atmospheres

T_t °R

A nomograph for the calculation of τ_1 appears in figure 17. The first turning line has been graduated to give directly the value τ_0, the time constant of a platinum wire at NACA standard sea-level conditions. The final result of the nomograph is τ_{Pt}, the time constant of a platinum wire at the given aerodynamic conditions. The time constant τ_1 for wire of any other material is obtained by multiplying τ_{Pt} by $(\rho_w c_w)/(\rho_{Pt} c_{Pt})$. The time constant τ in the presence of radiation is

$$\tau = \tau_1/(1 + 4\beta_1 \epsilon_w/T_g) \approx \tau_1/(1 + 4\bar{\beta}_1 \epsilon_w \epsilon_g/T_w) \tag{C9}$$

A tabulation of $(\rho_w c_w/\rho_{Pt} c_{Pt})$ for various thermocouple materials is given in table I. A tabulation of the arithmetically averaged values for commonly used thermocouple pairs is given in table II and is repeated for convenience in figure 17. The values appearing in table I were obtained from references 12 and 13 and are believed to represent the best values available. The data in figure 17 are thus sufficient for estimating τ_1 for any group of aerodynamic conditions that may normally be encountered in aircraft engine operation.

Steady-state conduction error. - Computation of the steady-state conduction error is conveniently accomplished by first evaluating the conduction parameter ηL. If the time constant τ has already been determined, ηL may be computed most simply by use of equation (11i) as

$$(\eta L)^2 = \frac{L^2}{\tau \kappa} \tag{C10}$$

The diffusivities of various materials are listed in table I.

If the time constant is not known, ηL may be computed by inserting equations (C5) and (C6) into equation (11c), yielding

$$(\eta_1 L)^2 = \frac{1.72\ k_{g,0}}{\mu_0^{1/2} r^{1/4} T_0^{0.435}} \frac{(M\ p)^{1/2} T_t^{0.18}}{D^{3/2} k_w} \left[\frac{\gamma}{\left(1 + \frac{\gamma-1}{2} M^2\right)}\right]^{1/4} L^2 \tag{C11a}$$

$$(\eta_1 L)^2 = 1.65\times10^{-3} \frac{(D\ M\ p)^{1/2} T_t^{0.18} (L/D)^2}{k_w\ (1 + 0.2M^2)^{1/4}} \tag{C11b}$$

where in equation (C11b) the units are

k_w Btu/ $\left[(\text{sec})(\text{ft})(^{\circ}\text{R})\right]$

D,L inches

p atmospheres

T_t $^{\circ}$R

A nomograph to facilitate calculation of $(\eta_1 L)^2$ is presented in figure 18. The final result of the nomograph is $(\eta_1 L)_{Pt}^2$, the value of $(\eta_1 L)^2$ for a platinum wire at the given aerodynamic conditions. The value of $(\eta_1 L)^2$ for a wire of different material is obtained by multiplying $(\eta_1 L)_{Pt}^2$ by (k_{Pt}/k_w). Values of (k_{Pt}/k_w) are presented in table I. A tabulation of the harmonically averaged values of k_{Pt}/k_w for commonly used thermocouple pairs is given in table II and is repeated for convenience in figure 18. The value of $(\eta L)^2$ is then

$$(\eta L)^2 = (\eta_1 L)^2 \left(1 + 4\beta_1 \epsilon_w / T_g\right) \approx (\eta_1 L)^2 \left(1 + 4\overline{\beta}_1 \epsilon_w \epsilon_g / T_w\right) \qquad \text{(C12)}$$

Finally, the conduction correction factor is

$$\psi_m = \text{sech}\ (\eta L/2) \qquad \text{(B8)}$$

and is given graphically in figure 19.

Corrections for differences between thermocouple wires. - If the two thermocouple materials differ appreciably in thermal conductivity, the computation of $(\eta_1 L)$ by formula (C11) should be carried out for each half of a wire, and then the effective $(\eta_1 L)$ for the pair of wires determined as indicated in part III of appendix B. If the two thermocouple materials are sufficiently alike, the effective thermal conductivity is the harmonic mean of the values for the individual materials.

If the two thermocouple materials differ in the values of ϵ_w and of $\rho_w c_w$, the effective values of these quantities are the arithmetic means of the respective values for the individual materials.

Time constant in presence of radiation and conduction. - The time constant in the presence of radiation and conduction is

$$\tau (1 - \psi_m) \approx \tau_1 \frac{1 - \psi_m}{(1 + 4\bar{\beta}_1 \epsilon_w \epsilon_g / T_w)} \tag{C12}$$

Gas temperature in presence of radiation and conduction errors. - The gas temperature when the thermocouple is subject to both radiation and conduction errors is

$$\left.\begin{aligned} T_g &= T_w - \bar{\beta}\left[(1-\alpha_{g,d})\left(\frac{T_d}{T_w}\right)^4 - (1-\epsilon_g)\right] - (T_b - T_w)\ \psi_m/(1-\psi_m) \\ &\approx T_w - \bar{\beta}\left[(1-\alpha_{g,d})\left(\frac{T_d}{T_w}\right)^4 - (1-\epsilon_g)\right] - (T_b - T_w)\ \psi_m \end{aligned}\right\} \tag{C13}$$

where T_b is the support temperature.

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TABLE I - PROPERTIES OF THERMOCOUPLE ELEMENTS AND ALLOYS

Material	Density ρ_w (lbm/ft^3) (a)	Specific heat, c_w $\left(\frac{\text{Btu}}{(\text{lbm})(^\circ\text{R})}\right)$ (a)	Thermal conductivity, k_w $\left(\frac{\text{Btu}}{(\text{ft})(\text{sec})(^\circ\text{R})}\right)$ (a)	Diffusivity, κ $\left(\frac{\text{in.}^2}{\text{sec}}\right)$	$\frac{\rho_w c_w}{\rho_{Pt} c_{Pt}}$	$\frac{k_{Pt}}{k_w}$
Platinum	1334	0.0324	0.0114_2	0.0381	1	1
Rhodium	774	.058	.0125	.040	1.05	.92
Platinum plus 13 percent rhodium	1261	.0357	[b] $.0048_4$	$.015_5$	1.05	2.38
Alumel	537	.124	[c] .0048	$.010_3$	1.55	2.3
Chromel	545	.106	[c] .0031	.0077	1.34	3.7
Constantan	553	.099	.0038	$.010_1$	1.27	2.9
Iron	491	.107	.0096	$.026_8$	1.22	1.19
Copper	555	.093	$.061_6$	.172	1.20	.19
Aluminum	169	.220	$.032_5$	.126	0.87	.35

[a]Evaluated for range 530° to 660° R except where noted.
[b]Evaluated at 530° R.
[c]Evaluated at 660° R.

TABLE II - PROPERTIES OF THERMOCOUPLE PAIRS

[Subscript A denotes positive material. This material is listed first in the pairs]

Material	$\frac{(\rho_w c_w)_A + (\rho_w c_w)_B}{2(\rho_{Pt} c_{Pt})}$	$\frac{k_{Pt}}{2}\left(\frac{1}{k_{w,A}} + \frac{1}{k_{w,B}}\right)$	$\left(\frac{k_{w,A}}{k_{w,B}}\right)^{0.5}$	$2\left(\frac{1}{\kappa_A} + \frac{1}{\kappa_B}\right)^{-1}$
Platinum plus 13 percent rhodium - platinum	1.02	1.69	1.54	0.0220
Chromel-alumel	1.45	3.0	.81	.0088
Chromel-constantan	1.31	3.3	.90	.0087
Iron-constantan	1.25	2.1	1.59	.0146

TABLE III - VALUES OF $\eta'L$

$\eta_A L$	$m_{A,B}$	$\eta'L$					
		$\eta_B L$					
		1	2	3	4	5	10
1	1.0	1.00	2.00	3.00	4.00	5.00	10.00
	.5	1.00	1.66	2.16	2.54	2.80	3.25
	0	1.00	1.40	1.67	1.85	1.97	2.15
	-.5	1.00	1.19	1.30	1.38	1.43	1.51
	-1.0	1.00	1.00	1.00	1.00	1.00	1.00
2	1.0	1.00	2.00	3.00	4.00	5.00	10.00
	.5	1.19	2.00	2.69	3.25	3.68	4.52
	0	1.40	2.00	2.43	2.73	2.94	3.28
	-.5	1.66	2.00	2.21	2.33	2.41	2.54
	-1.0	2.00	2.00	2.00	2.00	2.00	2.00
3	1.0	1.00	2.00	3.00	4.00	5.00	10.00
	.5	1.30	2.20	3.00	3.70	4.27	5.55
	0	1.67	2.43	3.00	3.44	3.75	4.30
	-.5	2.16	2.69	3.00	3.21	3.34	3.55
	-1.0	3.00	3.00	3.00	3.00	3.00	3.00
4	1.0	1.00	2.00	3.00	4.00	5.00	10.00
	.5	1.38	2.33	3.21	4.00	4.70	6.48
	0	1.85	2.73	3.44	4.00	4.44	5.28
	-.5	2.54	3.25	3.70	4.00	4.21	4.54
	-1.0	4.00	4.00	4.00	4.00	4.00	4.00
5	1.0	1.00	2.00	3.00	4.00	5.00	10.00
	.5	1.43	2.41	3.34	4.21	5.00	7.33
	0	1.97	2.94	3.75	4.44	5.00	6.23
	-.5	2.80	3.68	4.27	4.70	5.00	5.52
	-1.0	5.00	5.00	5.00	5.00	5.00	5.00
10	1.0	1.00	2.00	3.00	4.00	5.00	10.00
	.5	1.51	2.55	3.55	4.54	5.52	10.00
	0	2.15	3.28	4.30	5.28	6.23	10.00
	-.5	3.25	4.52	5.55	6.48	7.33	10.00
	-1.0	10.00	10.00	10.00	10.00	10.00	10.00

TABLE IV - VALUES OF $\eta''L$

$\eta_Q L$	$\frac{m_P}{m_Q}$	$\eta_P(L-L')$ = 1	2	3	4	5	10
1	0.01	1.45	2.31	3.27	4.26	5.25	10.2
	.02	1.45	2.32	3.28	4.27	5.26	10.2
	.05	1.47	2.34	3.30	4.29	5.29	10.2
	.10	1.51	2.38	3.35	4.34	5.33	10.3
	.20	1.57	2.46	3.43	4.42	5.42	10.4
	.50	1.75	2.68	3.66	4.66	5.66	10.6
	1.0	2.00	3.00	4.00	5.00	6.00	11.0
2	.01	2.31	3.04	3.94	4.90	5.89	10.8
	.02	2.32	3.05	3.95	4.92	5.90	10.9
	.05	2.34	3.09	3.99	4.96	5.94	10.9
	.10	2.38	3.15	4.06	5.03	6.02	11.0
	.20	2.46	3.26	4.19	5.17	6.15	11.1
	.50	2.68	3.57	4.53	5.52	6.51	11.5
	1.0	3.00	4.00	5.00	6.00	7.00	12.0
3	.01	3.27	3.94	4.80	5.76	6.74	11.7
	.02	3.28	3.95	4.82	5.77	6.75	11.7
	.05	3.30	3.99	4.87	5.82	6.81	11.8
	.10	3.35	4.06	4.95	5.91	6.89	11.9
	.20	3.43	4.19	5.09	6.06	7.05	12.0
	.50	3.66	4.53	5.48	6.47	7.46	12.4
	1.0	4.00	5.00	6.00	7.00	8.00	13.0
4	.01	4.26	4.90	5.76	6.70	7.68	12.6
	.02	4.27	4.92	5.77	6.72	7.70	12.7
	.05	4.29	4.96	5.82	6.78	7.75	12.7
	.10	4.34	5.03	5.91	6.86	7.84	12.8
	.20	4.42	5.17	6.06	7.03	8.01	13.0
	.50	4.66	5.52	6.47	7.45	8.44	13.4
	1.0	5.00	6.00	7.00	8.00	9.00	14.0
5	.01	5.25	5.89	6.74	7.68	8.66	13.6
	.02	5.26	5.90	6.75	7.70	8.67	13.6
	.05	5.28	5.94	6.81	7.75	8.73	13.7
	.10	5.33	6.02	6.89	7.84	8.82	13.8
	.20	5.42	6.15	7.05	8.01	8.99	14.0
	.50	5.65	6.51	7.46	8.44	9.43	14.4
	1.0	6.00	7.00	8.00	9.00	10.00	15.0
10	.01	10.2	10.8	11.7	12.6	13.6	18.6
	.02	10.2	10.9	11.7	12.7	13.6	18.6
	.05	10.2	10.9	11.8	12.7	13.7	18.7
	.10	10.3	11.0	11.9	12.8	13.8	18.8
	.20	10.4	11.1	12.0	13.0	14.0	18.9
	.50	10.6	11.5	12.4	13.4	14.4	19.4
	1.0	11.0	12.0	13.0	14.0	15.0	20.0

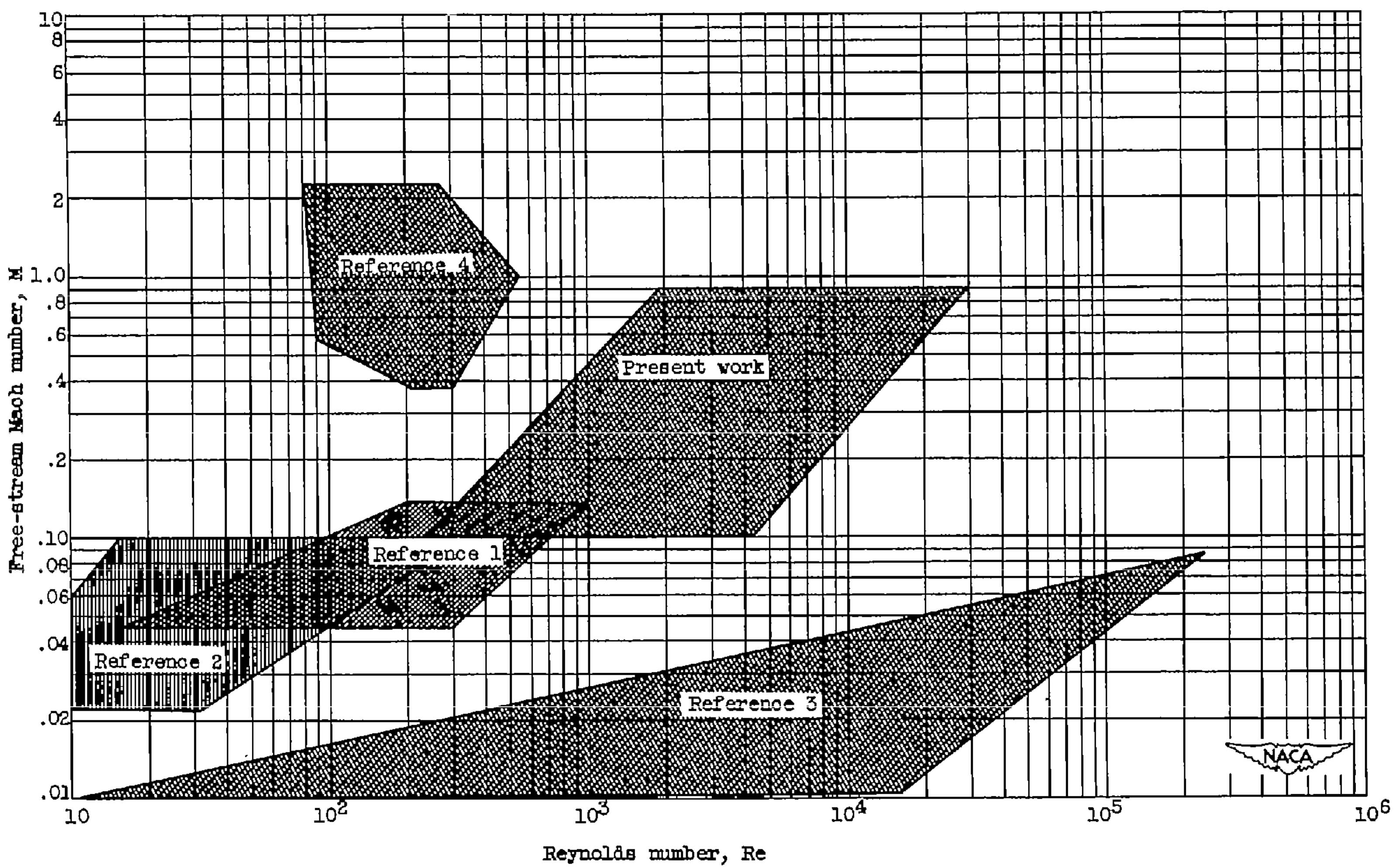
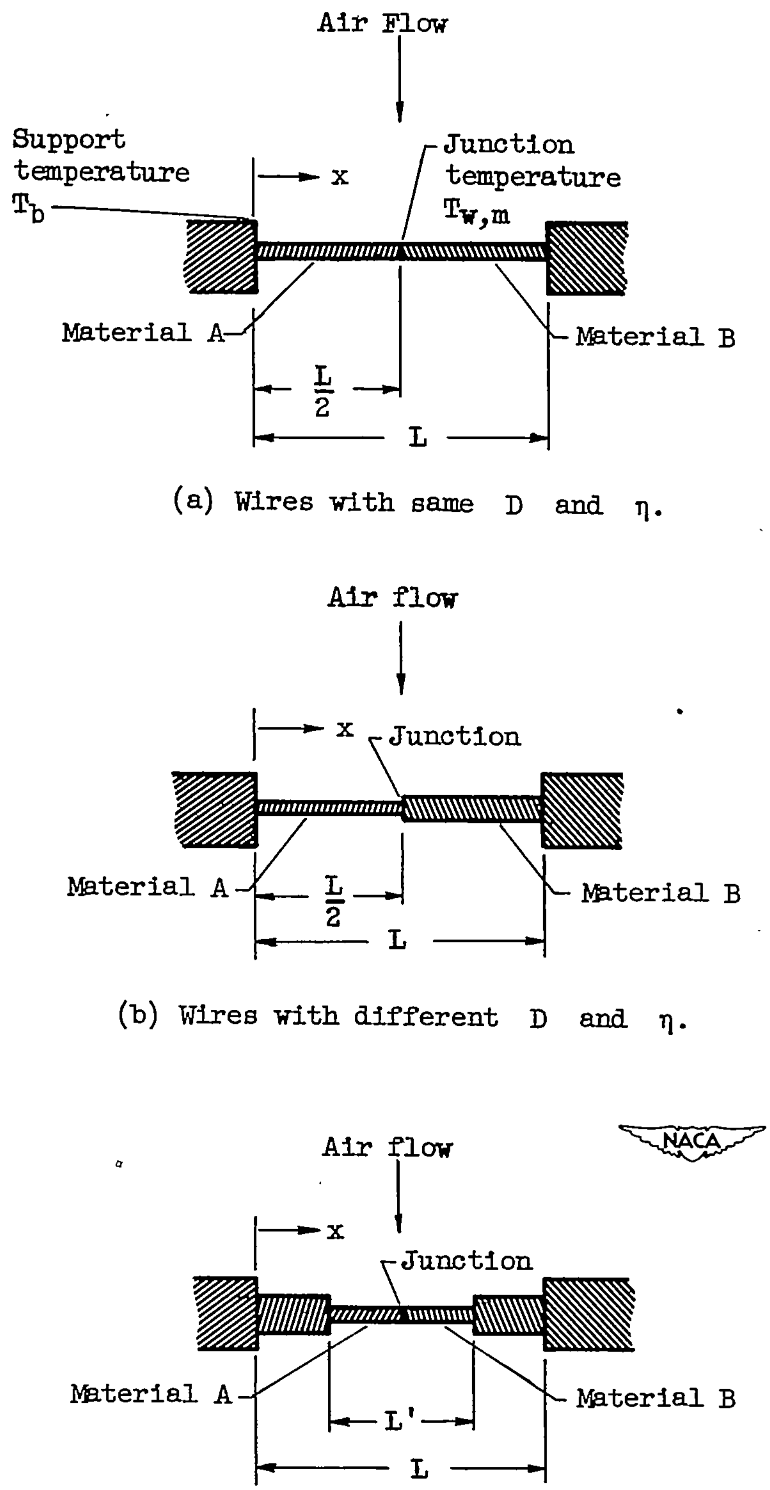


Figure 1. - Comparison of Mach and Reynolds number ranges of present and previous work.

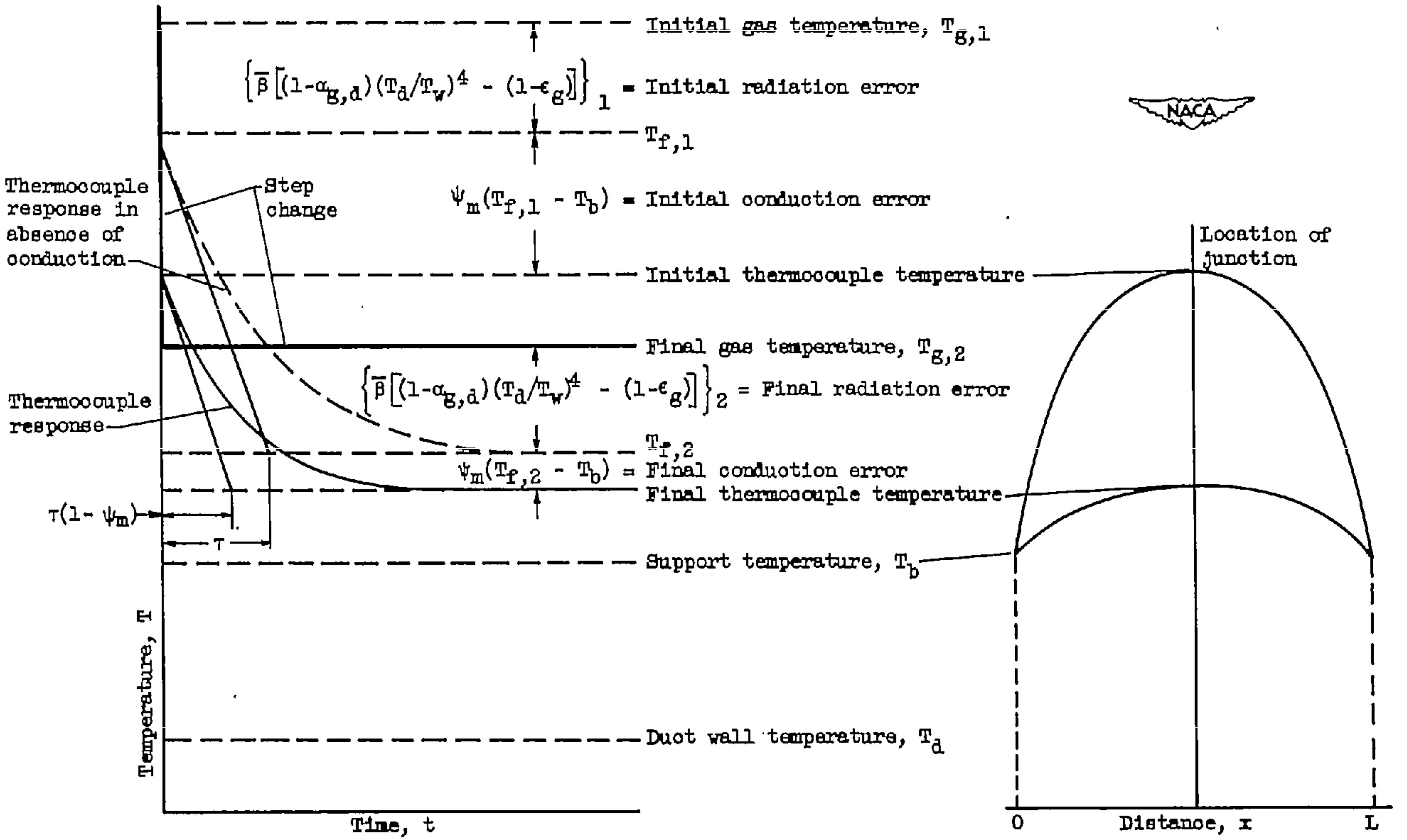


(a) Wires with same D and η.

(b) Wires with different D and η.

(c) Wires supported between intermediate diameter struts.

Figure 2. - Basis thermocouple designs (developed views).



(a) Temperature change with time.

(b) Temperature change with distance along wire.

Figure 3. - Variation of temperature with time and distance along simple wire where both materials are of same diameter and thermal conductivity.

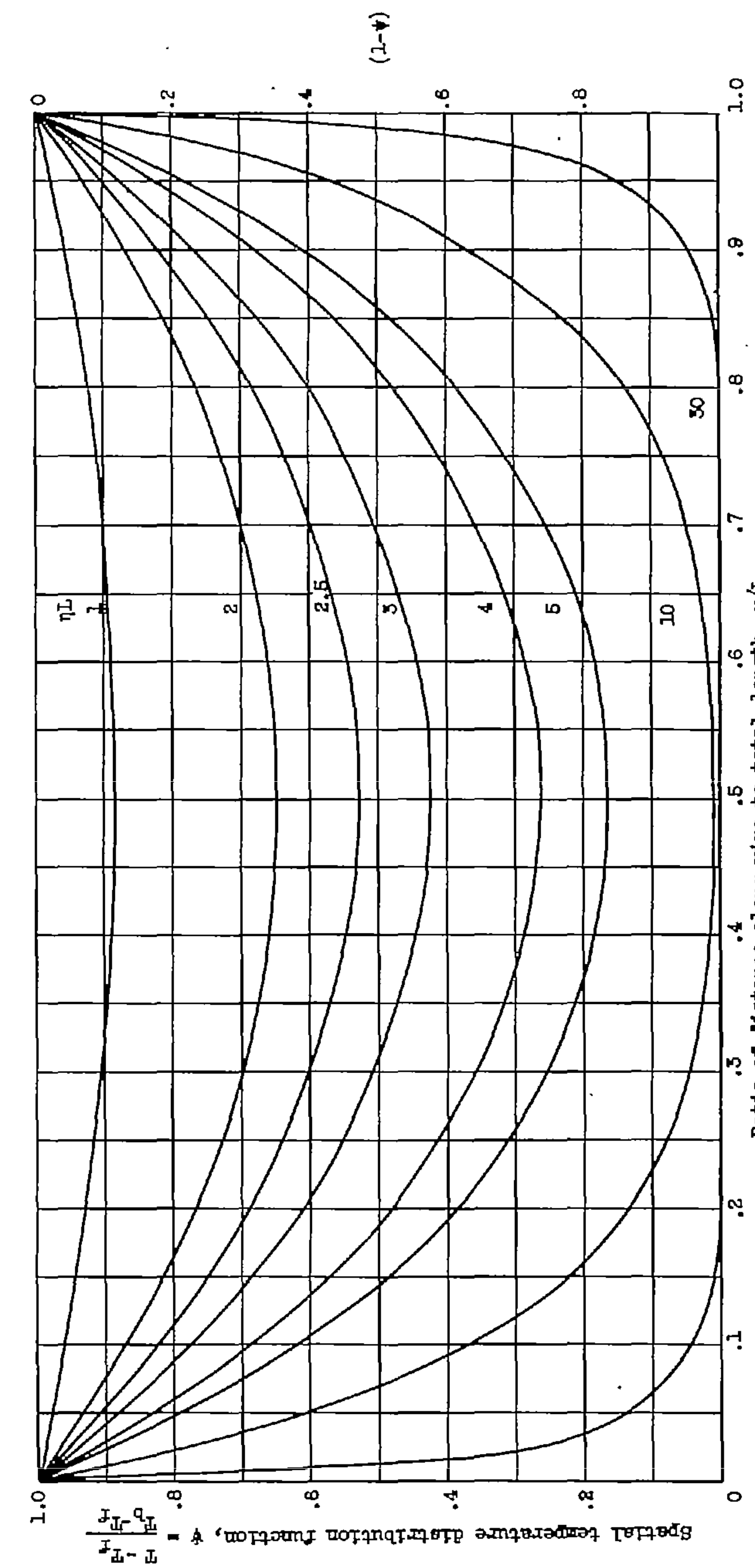


Figure 4. - Temperature distribution along simple wire in steady state.

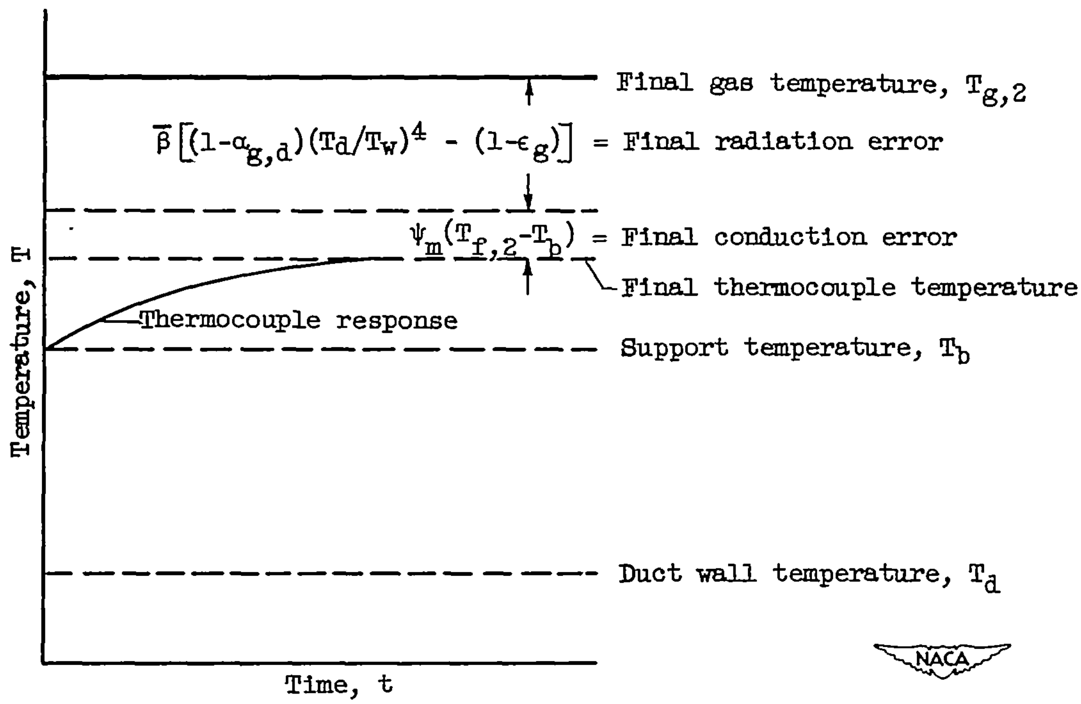


Figure 5. - Response to step change from support temperature. Thermocouple initially at support temperature.

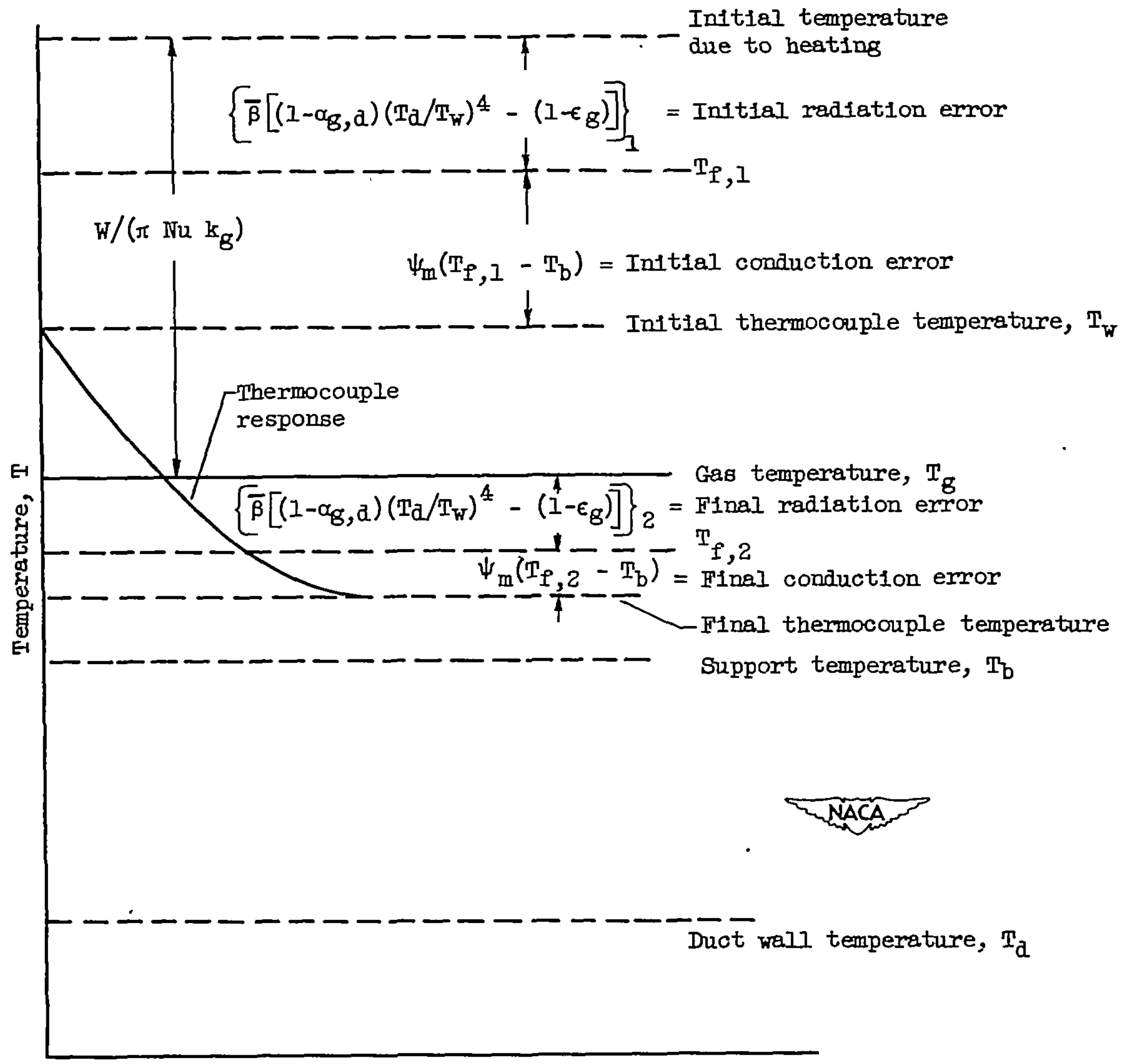


Figure 6. - Response to step change with wire initially heated to temperature greater than gas temperature.

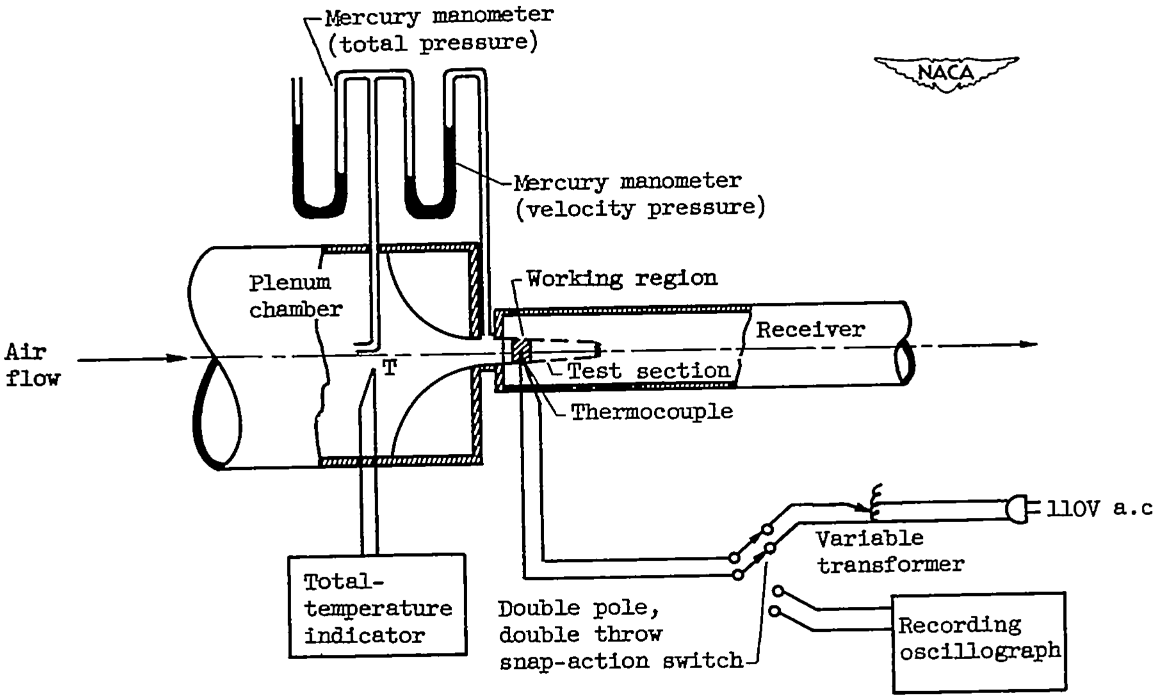
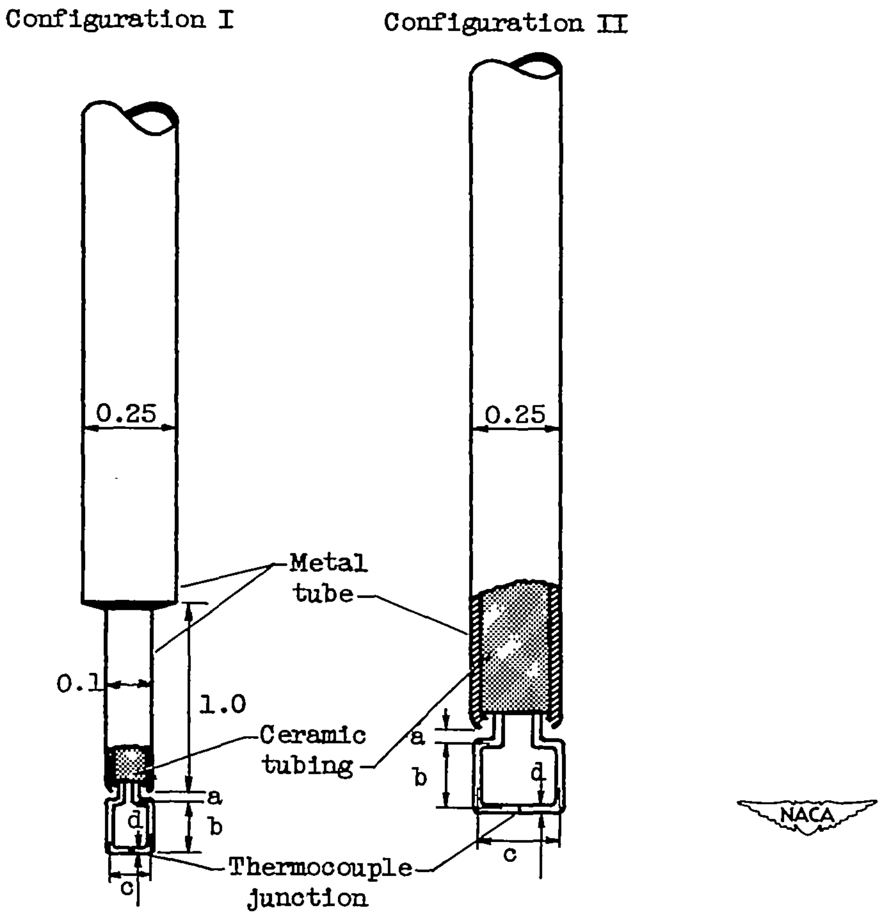


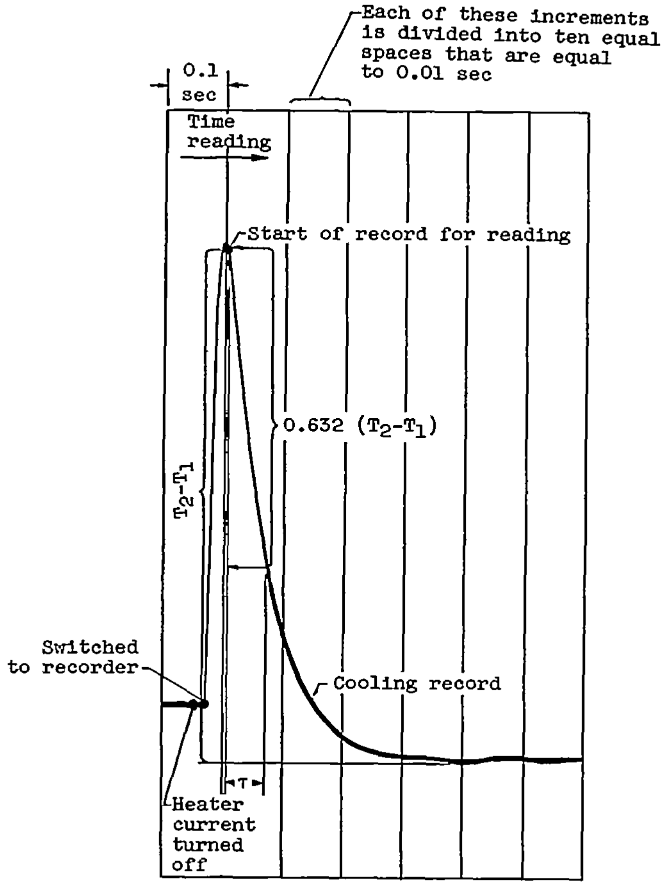
Figure 7. - Setup and instrumentation for determination of time response of thermocouples.



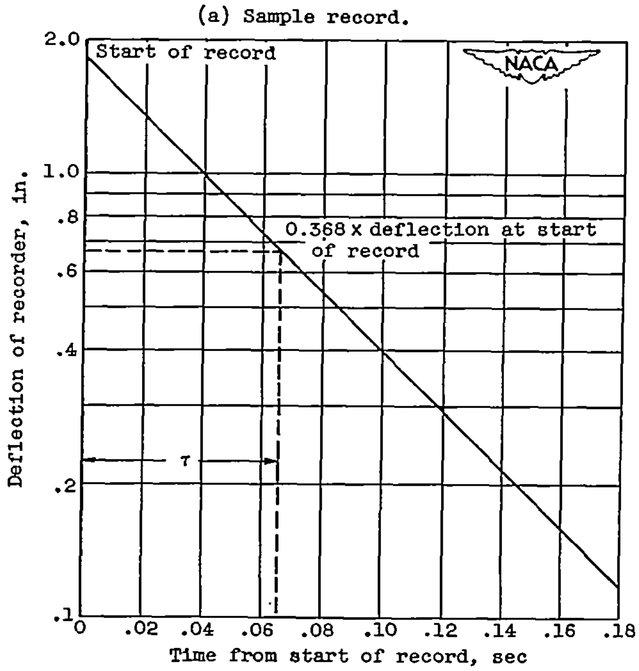
Probe	Thermocouple material	a	b	c	d[1]	Configuration
A	Chromel-constantan	0.02	0.15	0.05	0.0085	I
B	--------do.----------	.04	.30	.10	.0176	I
C	--------do.----------	.10	.50	.25	.0554	II
D	Iron-constantan	.04	.30	.10	.0160	I
E	Chromel-alumel	.04	.30	.10	.0155	I
F	Platinum plus 13-percent rhodium - platinum	.04	.30	.10	.0174	I
G	Chromel-constantan	----	----	---	.0176	Straight wire

[1]Dimension d is the average of eight measurements.

Figure 8. - Thermocouples used for test purposes. Air flow perpendicular to plane of thermocouple loops. (All dimensions are in inches.)

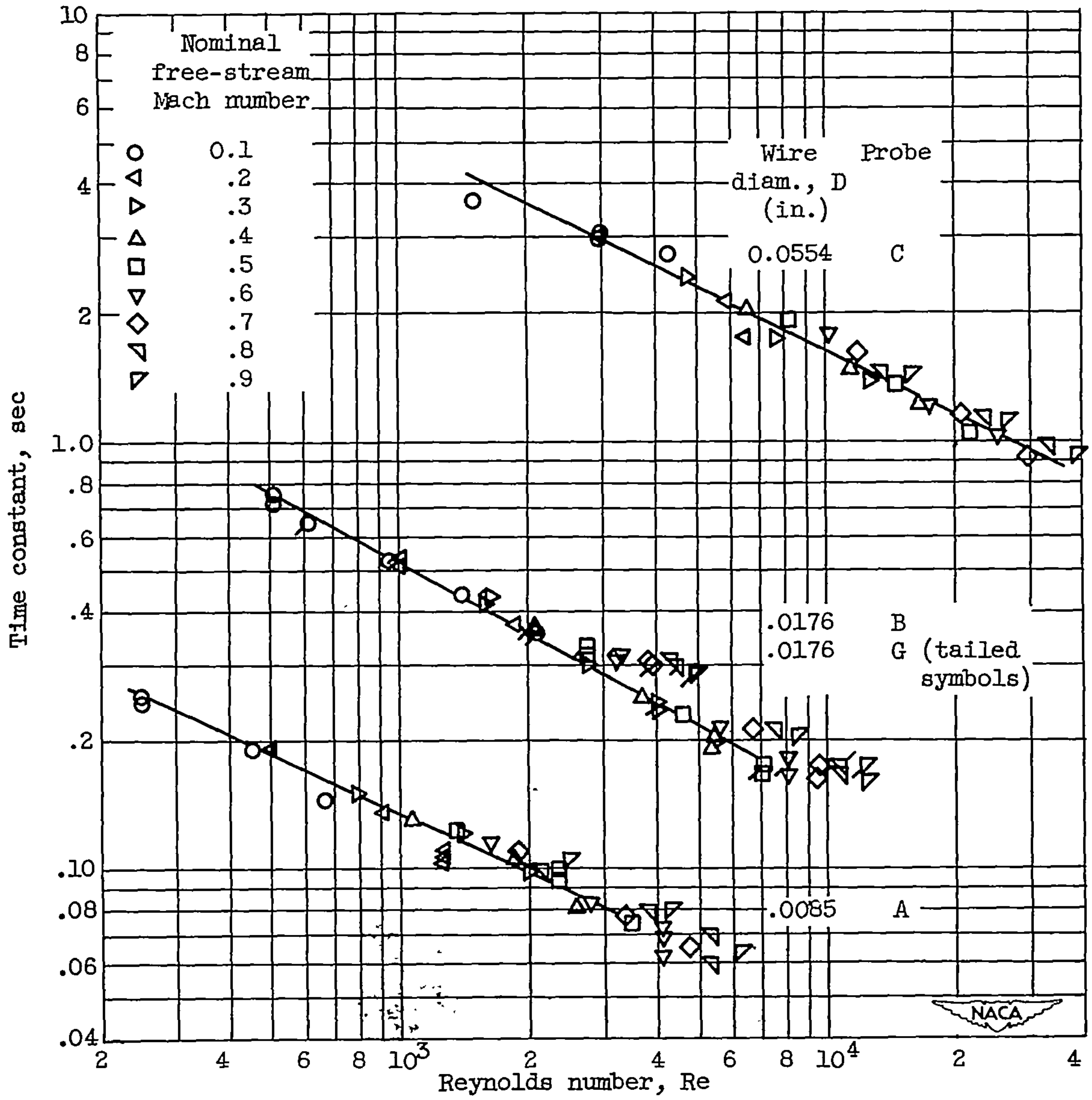


(a) Sample record.



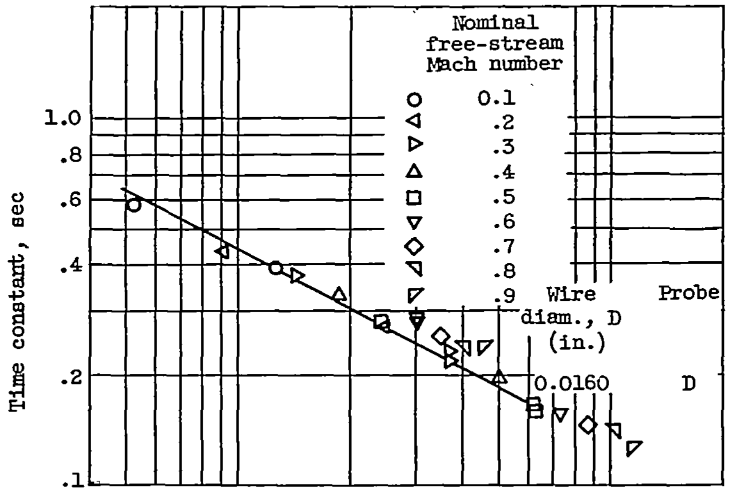
(b) Cooling curve plotted to show first-order relation.

Figure 9. - Time-constant determination.

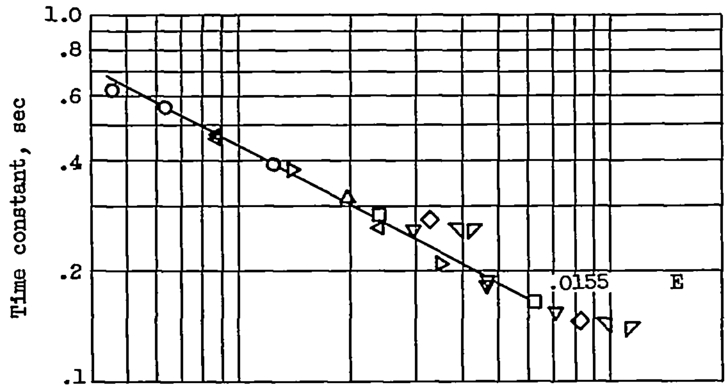


(a) Chromel-constantan thermocouple.

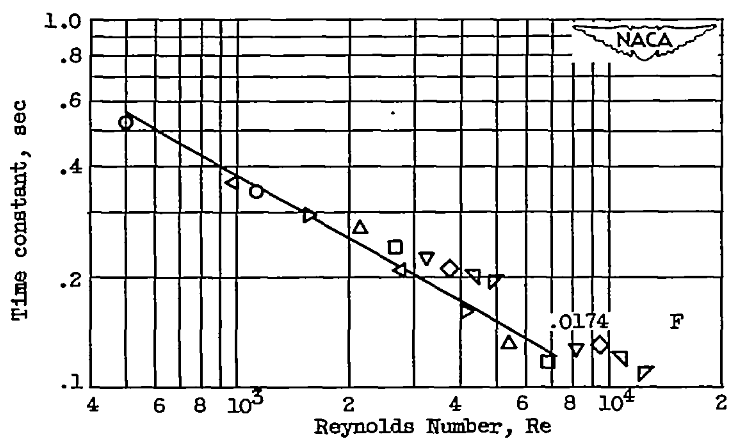
Figure 10. - Time constants of thermocouples with Reynolds number based on static temperature. Lines drawn through data points for M<0.5.



(b) Iron-constantan thermocouple.

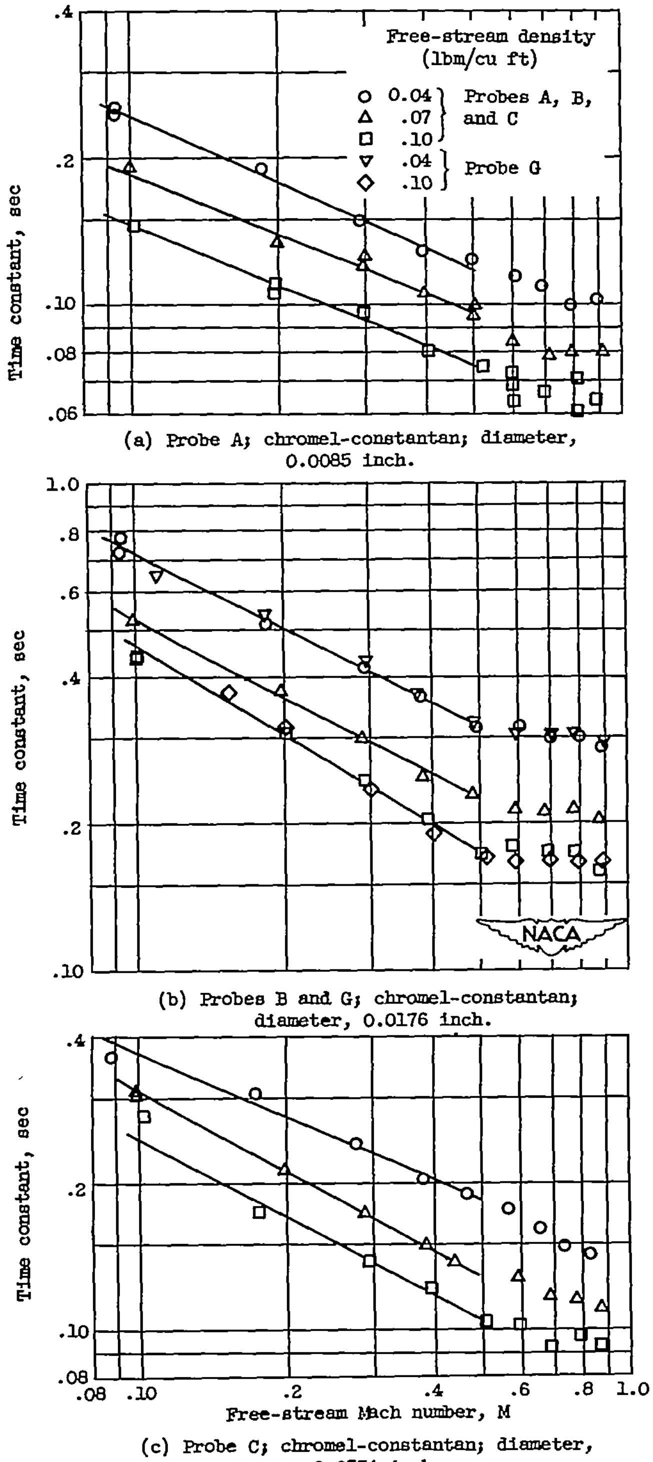


(c) Chromel-alumel thermocouple.



(d) Platinum plus 13-percent rhodium - platinum thermocouple.

Figure 10. - Concluded. Time constants of thermocouples with Reynolds number based on static temperature. Lines drawn through data points for M<0.5.

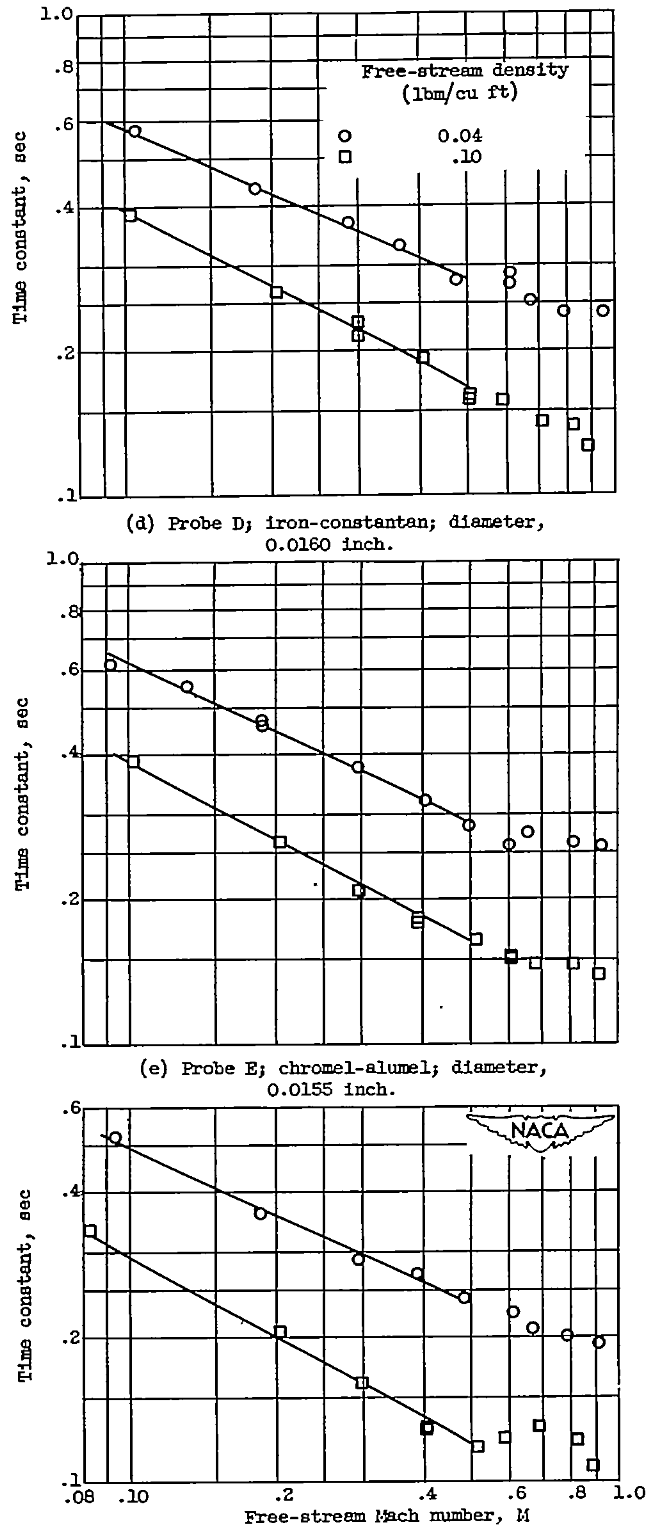


(a) Probe A; chromel-constantan; diameter, 0.0085 inch.

(b) Probes B and G; chromel-constantan; diameter, 0.0176 inch.

(c) Probe C; chromel-constantan; diameter, 0.0554 inch.

Figure 11. - Time constants of seven thermocouples.

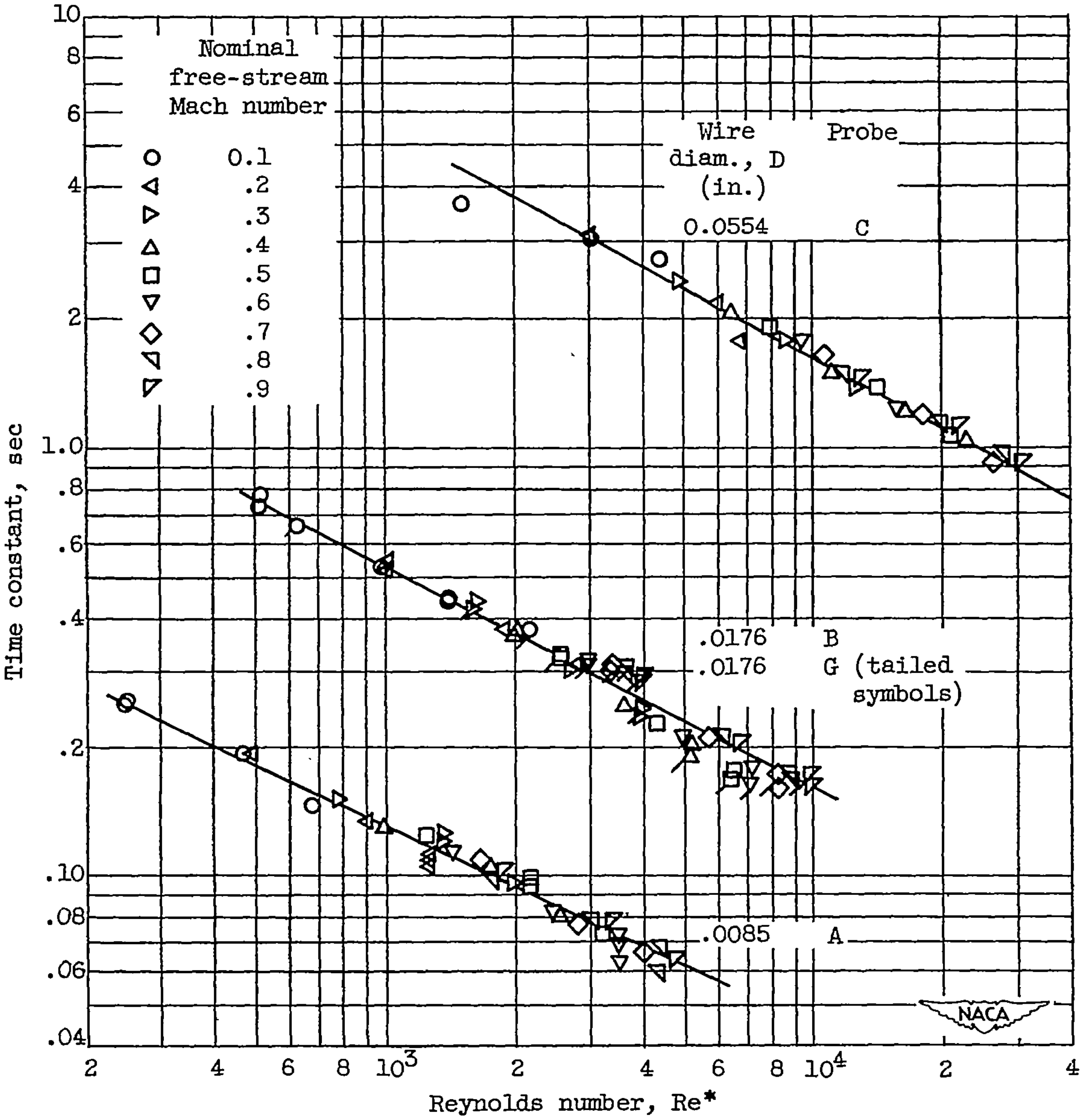


(d) Probe D; iron-constantan; diameter, 0.0160 inch.

(e) Probe E; chromel-alumel; diameter, 0.0155 inch.

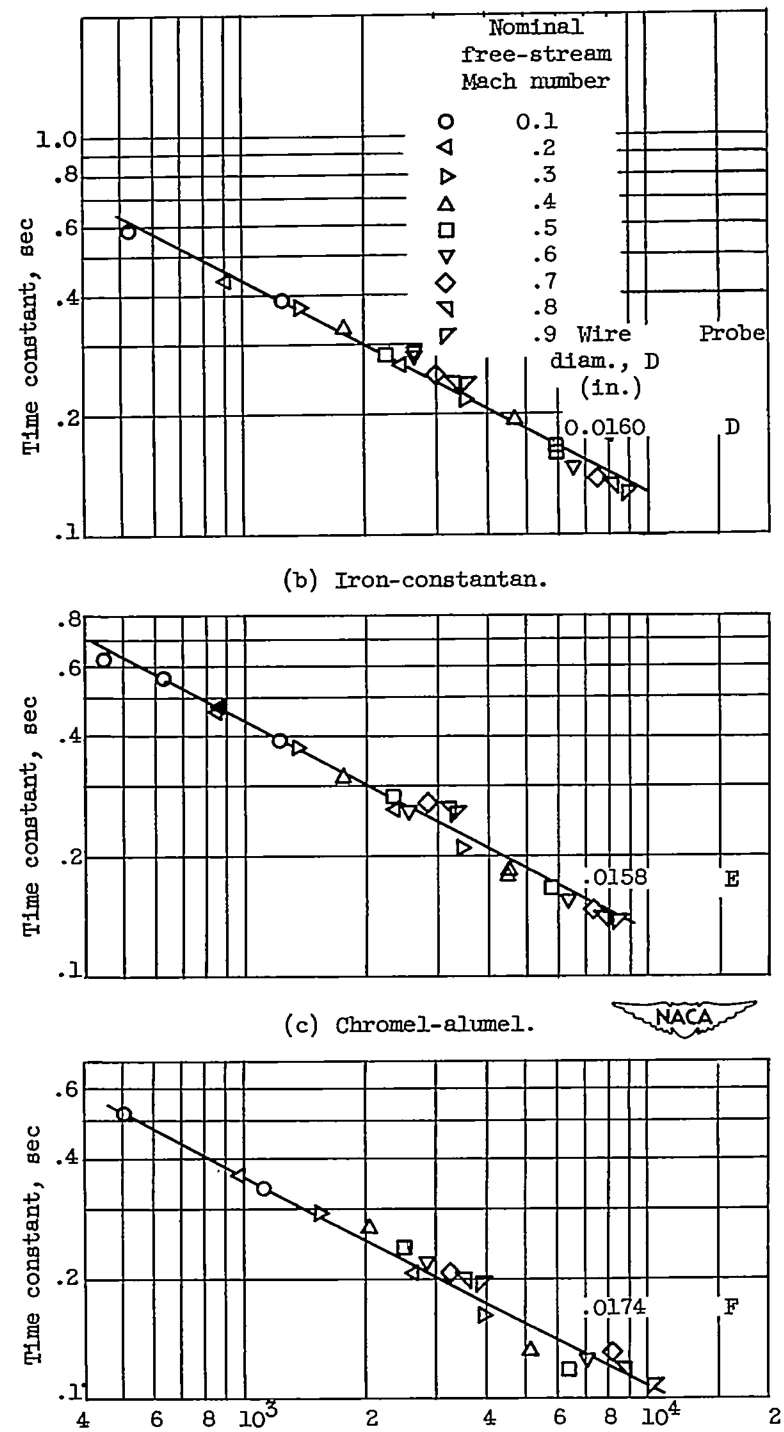
(f) Probe F; platinum plus 13-percent rhodium - platinum; diameter, 0.0174 inch.

Figure 11. - Concluded. Time constants of seven thermocouples.



(a) Chromel-constantan.

Figure 12. - Time constants of thermocouples with Reynolds number based on total temperature.



(b) Iron-constantan.

(c) Chromel-alumel.

(d) Platinum plus 13-percent rhodium - platinum.

Figure 12. - Concluded. Time constants of thermocouples with Reynolds number based on total temperature.

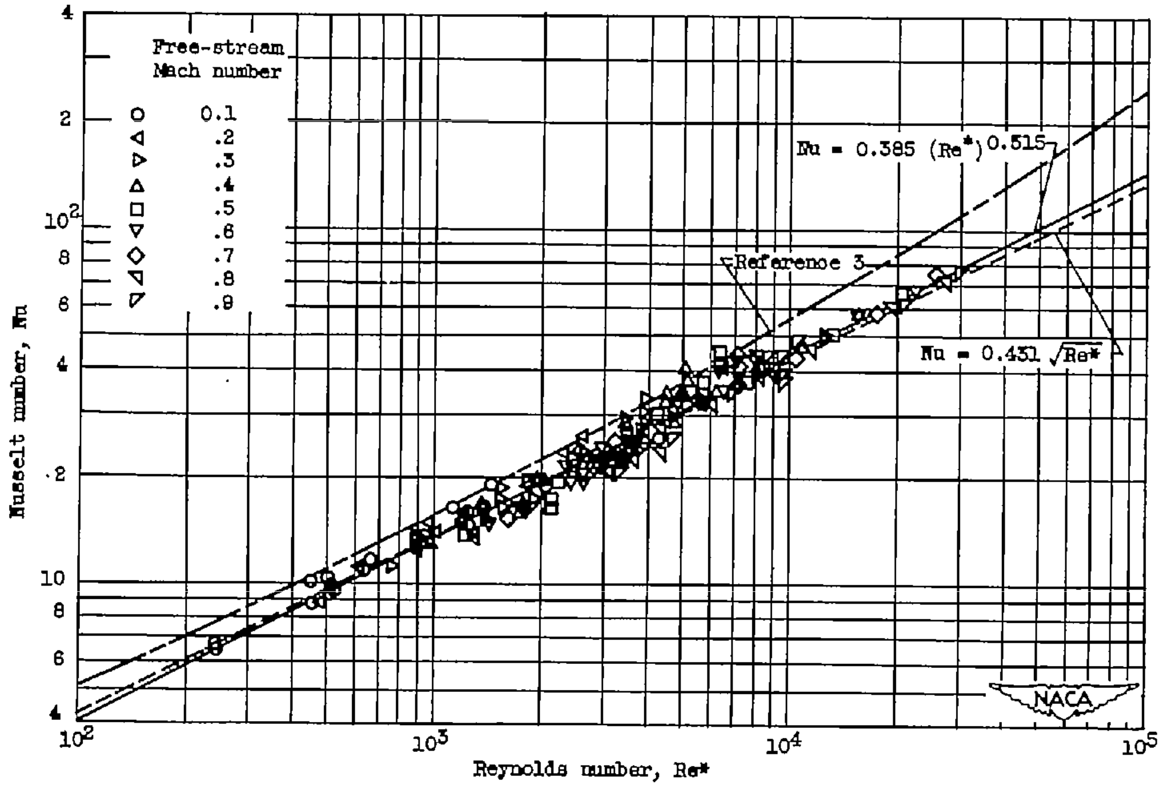


Figure 13. - Correlation of Nusselt number with Reynolds number for air for all data. Constant Prandtl number.

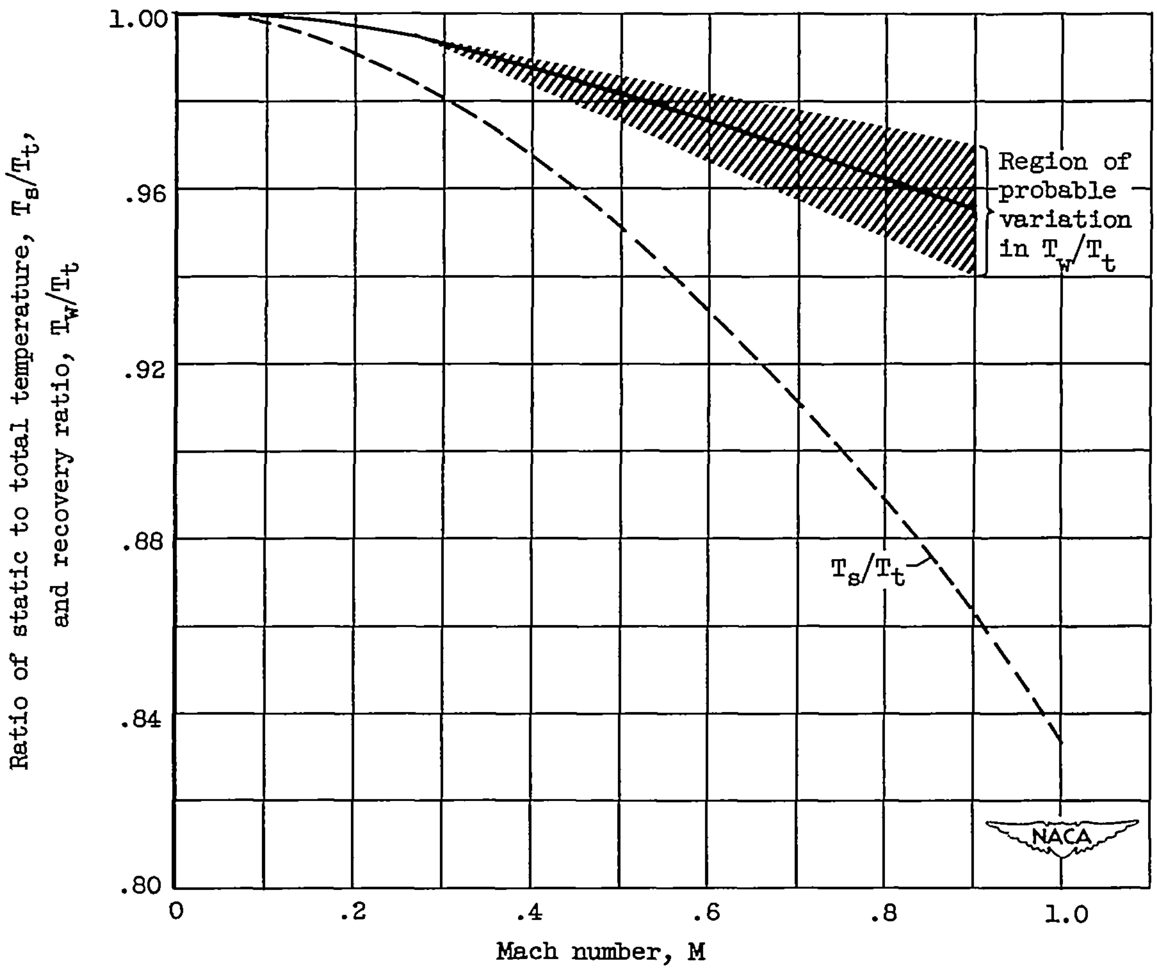


Figure 14. - Recovery ratio of bare-wire thermocouple. Average of previously accumulated data.

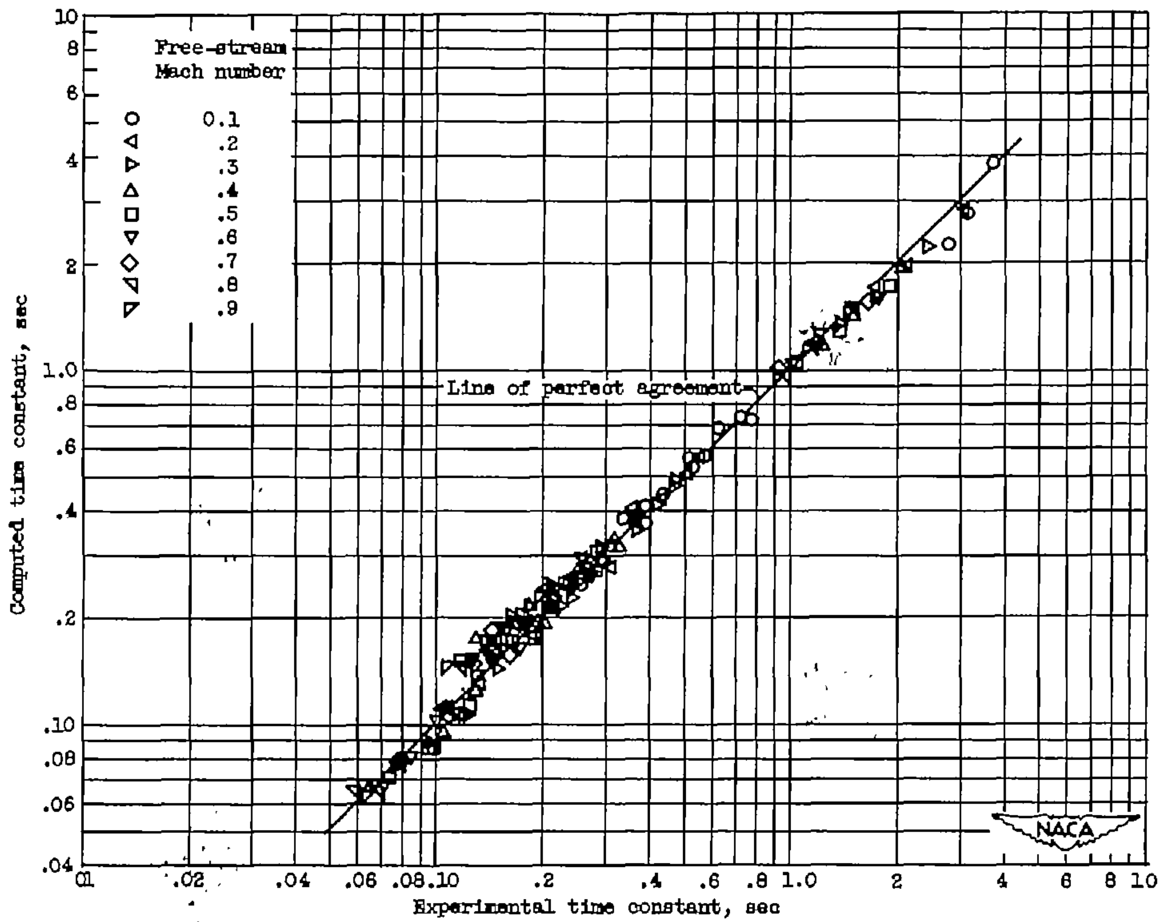
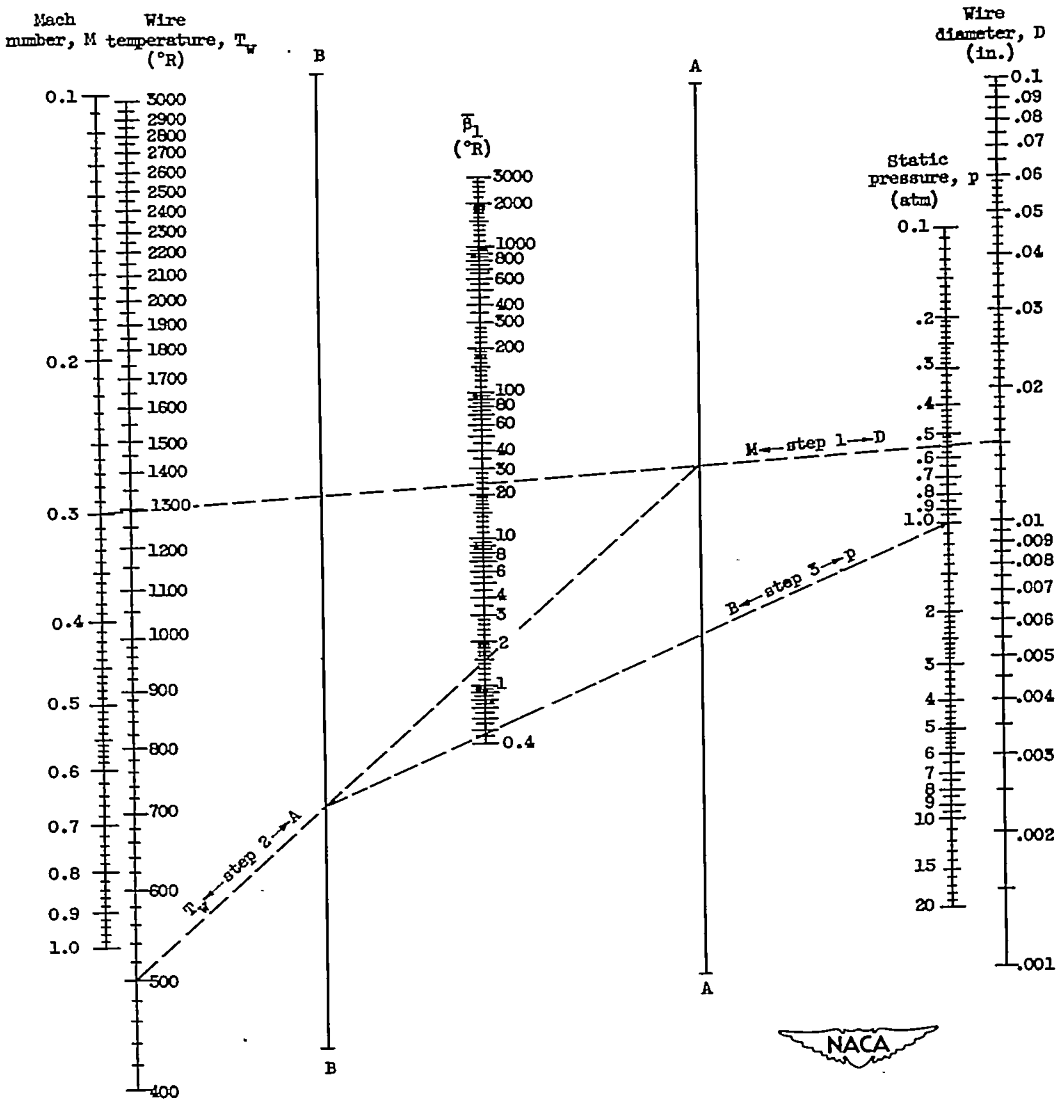


Figure 15. - Comparison of computed time constant with experimental time constant.



Given: T_w = 500° R
p = 1 atm
M = 0.3
D = 0.015 in.

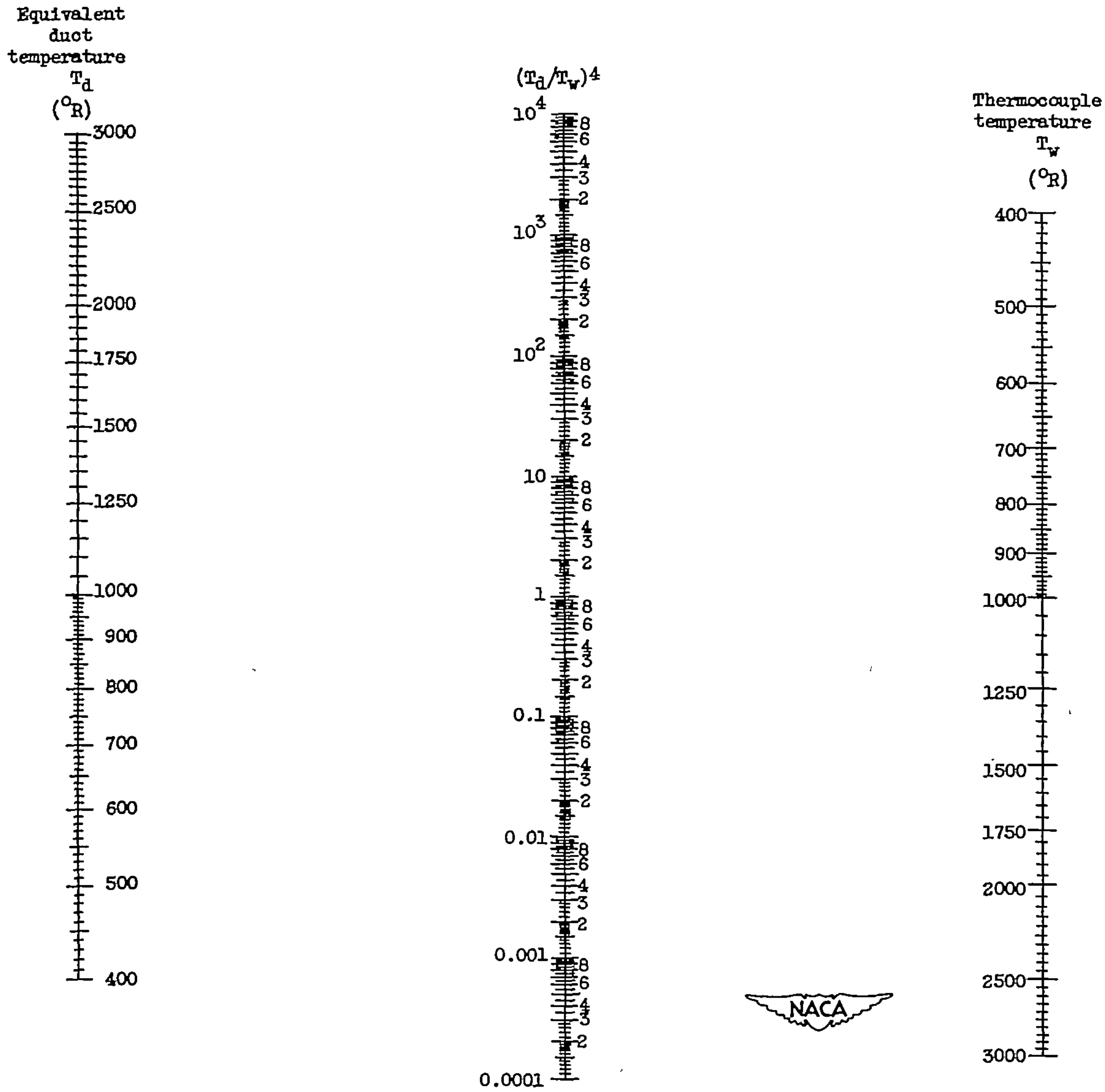
Procedure: Connect M to D and find intersection A.
Connect A to T_w and find intersection B.
Connect B to p and find answer $\bar{\beta}_1$.

Answer: $\bar{\beta}_1$ = 0.46° R

(a) Nomograph for computing $\bar{\beta}_1$.

Figure 16. - Nomographs for determining radiation error.

(A 10- by 12-in. print of this fig. is attached.)



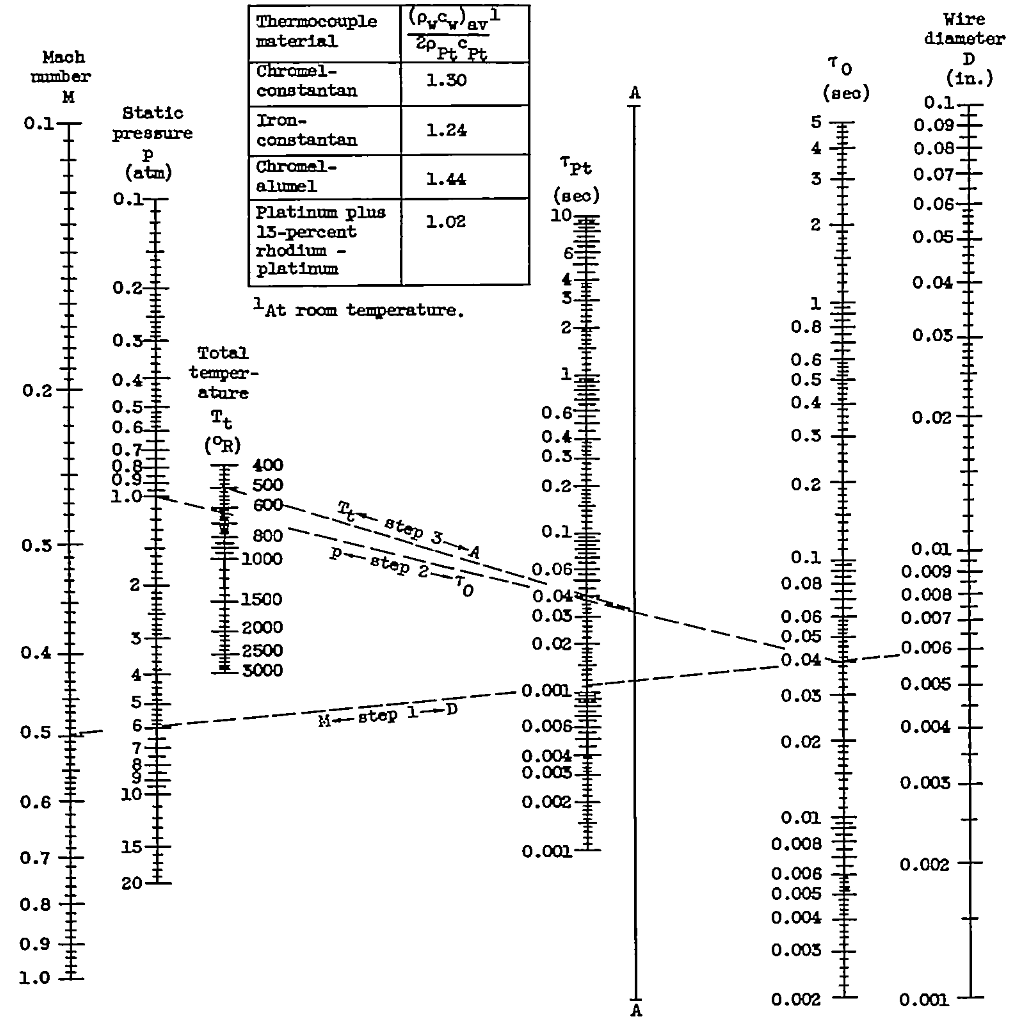
(b) Nomograph for computing $(T_d/T_w)^4$.

Figure 16. - Concluded. Nomographs for determining radiation error.

(A 9- by 10-in. print of this fig. is attached.)

Thermocouple material	$\frac{(\rho_w c_w)_{av}}{2\rho_{Pt} c_{Pt}}$ [1]
Chromel-constantan	1.30
Iron-constantan	1.24
Chromel-alumel	1.44
Platinum plus 13-percent rhodium - platinum	1.02

[1]At room temperature.



Given: M = 0.5
D = 0.006 in.
p = 1 atm
T_t = 500° R
Chromel-alumel

Procedure: Connect M to D and find intersection τ_0
Connect τ_0 to p and find intersection A.
Connect A to T_t and find intersection τ_{Pt}.
Multiply τ_{Pt} by number in table to find τ_1.

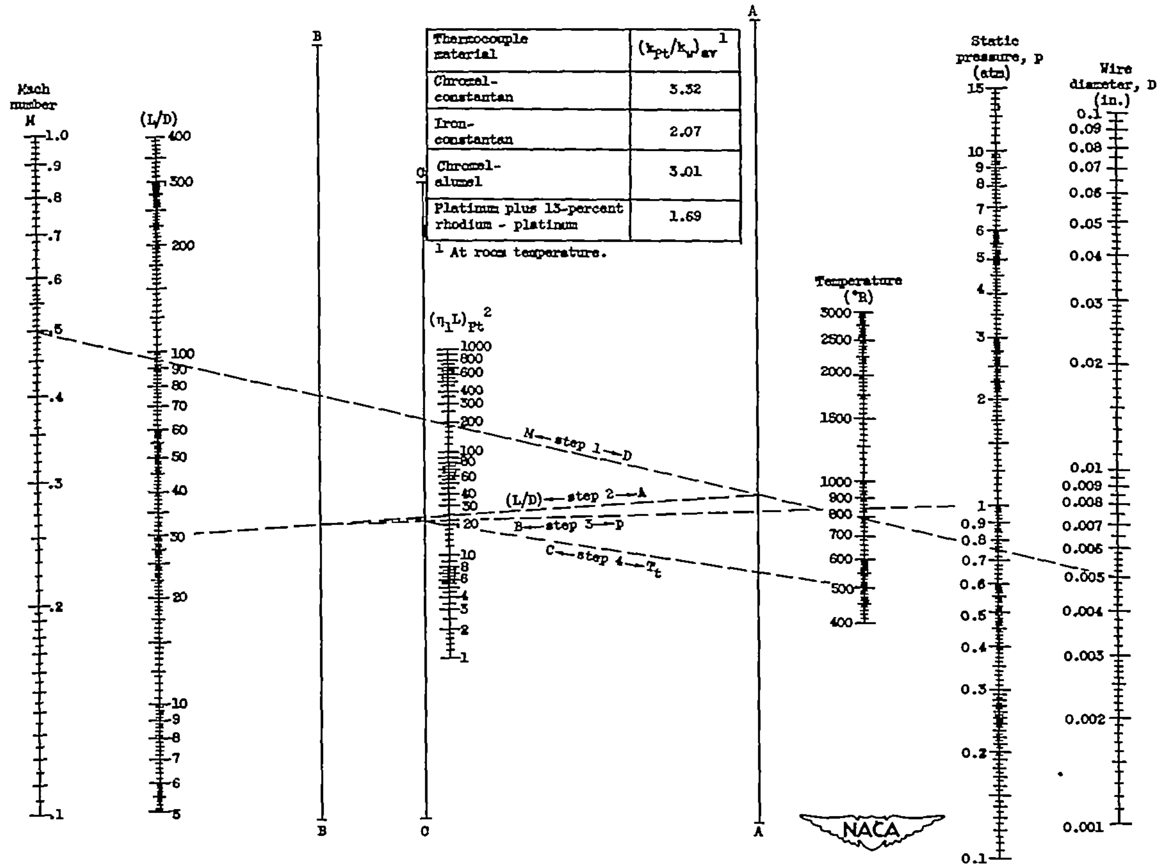
Answer: τ_{Pt} = 0.04 sec

$$\tau_1 = \tau_{Pt}\left[\frac{(\rho_w c_w)_{av}}{2\rho_{Pt} c_{Pt}}\right]$$

τ_1 = 0.057 sec

Figure 17. - Nomograph for computing τ_1.

(A 11- by 12-in. print of this fig. is attached.)



Thermocouple material	$(k_{Pt}/k_w)_{av}$ [1]
Chromel-constantan	3.32
Iron-constantan	2.07
Chromel-alumel	3.01
Platinum plus 13-percent rhodium - platinum	1.69

[1] At room temperature.

Given: $M = 0.5$
$D = 0.005$ in.
$L/D = 30$
$p = 1$ atm
$T_t = 500°$ R
Chromel-alumel

Procedure: Connect M to D and find intersection A.
Connect A to L/D and find intersection B.
Connect B to p and find intersection C.
Connect C to T_t and find intersection $(\eta_1 L)_{Pt}^2$.
Multiply $(\eta_1 L)_{Pt}^2$ by value in table to find $(\eta_1 L)^2$.

Answer: $(\eta_1 L)_{Pt}^2 = 20$

$(\eta_1 L)^2 = (\eta_1 L)_{Pt}^2 \left(\frac{k_{Pt}}{k_w}\right)$

$(\eta_1 L)^2 = 60$

Figure 18. - Nomograph for computing $(\eta_1 L)^2$.

(A 12- by 13-in. print of this fig. is attached.)

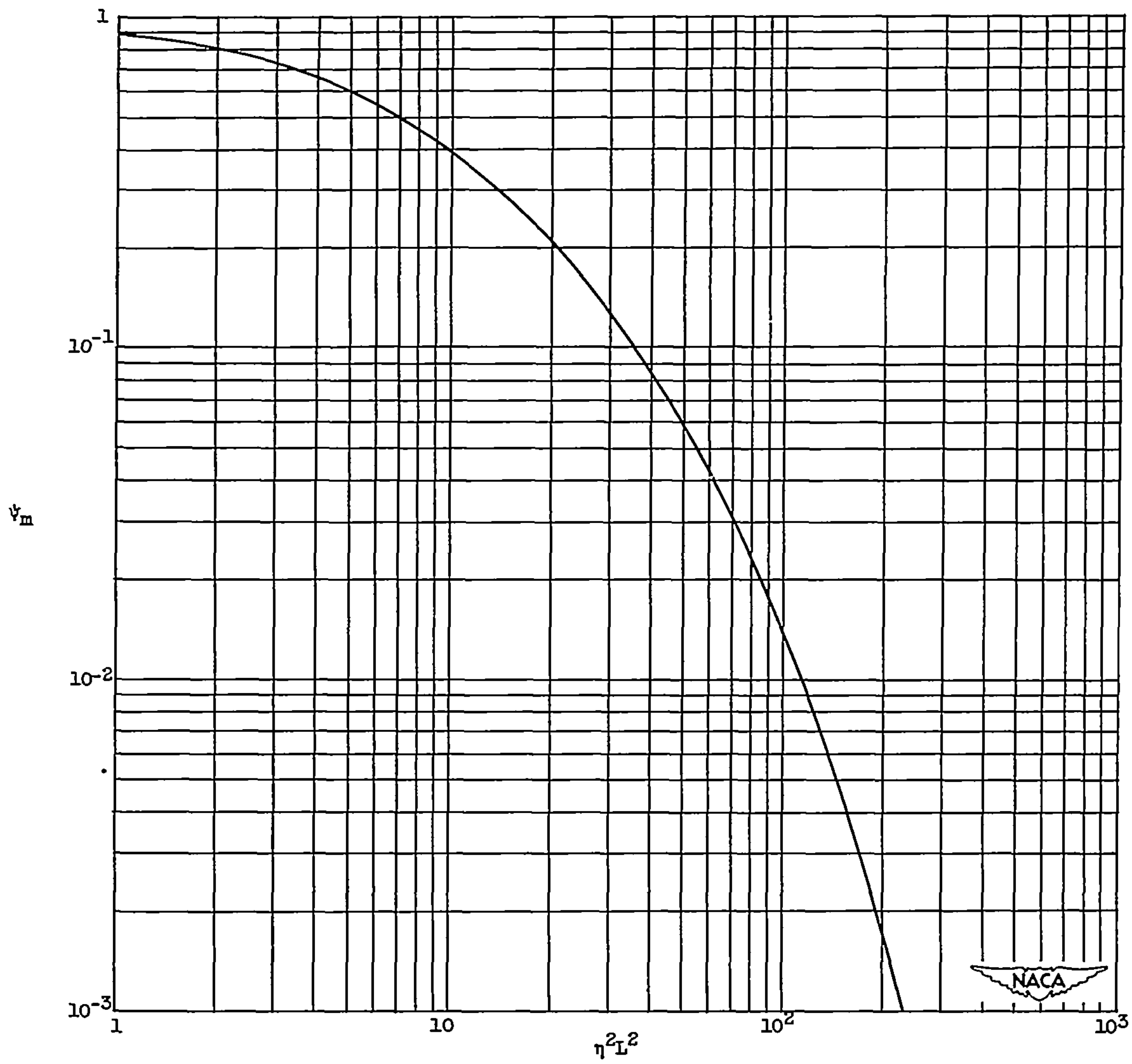


Figure 19. - Conduction correction factor, $\psi_m = \text{sech}\,\frac{\eta L}{2}$.

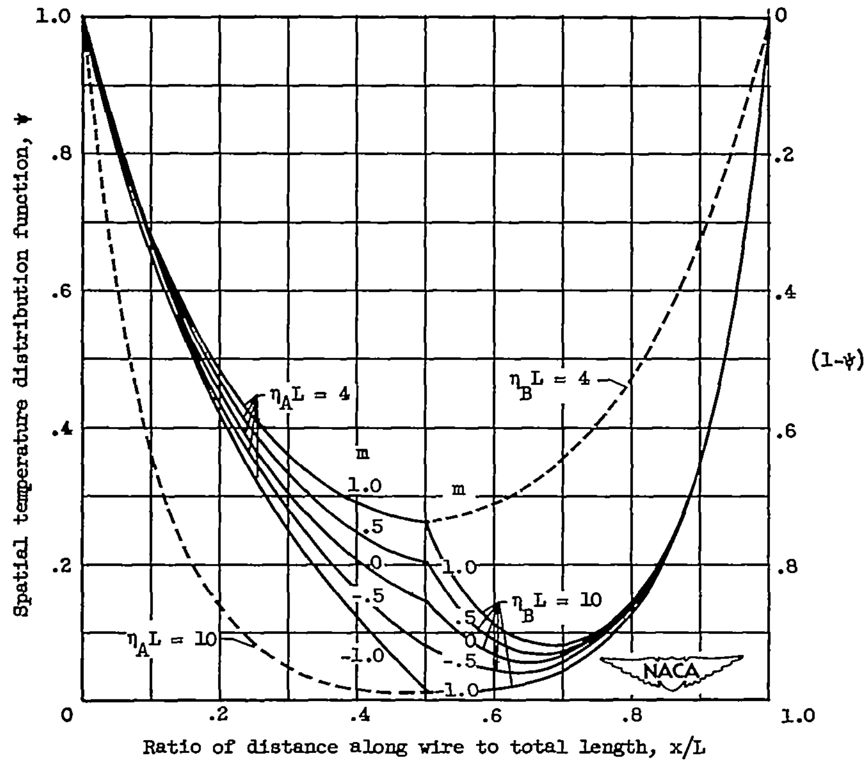


Figure 20. - Temperature distribution along thermocouple consisting of two wires of different thermal conductivity or diameter.

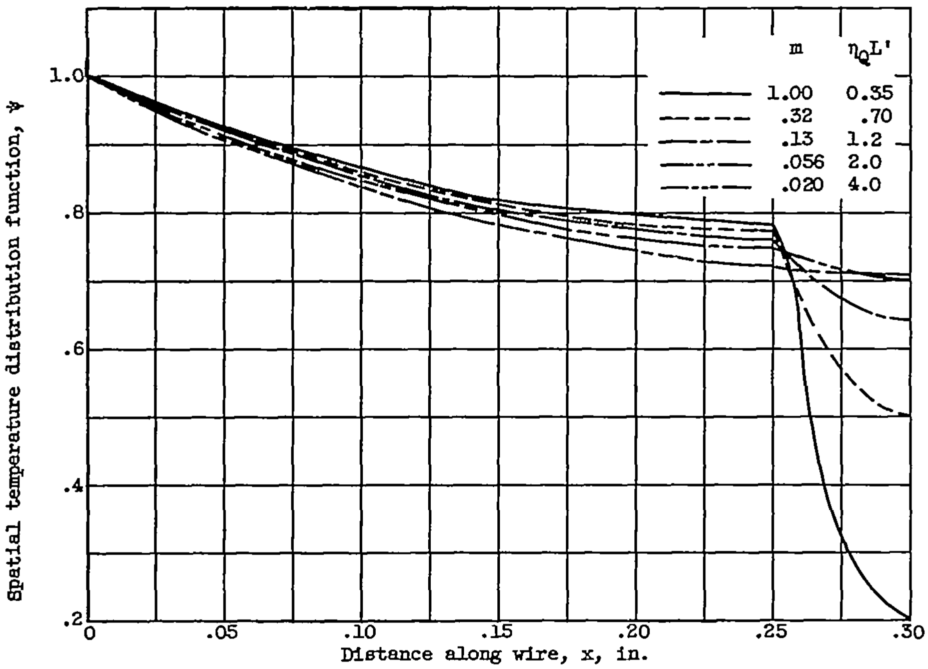
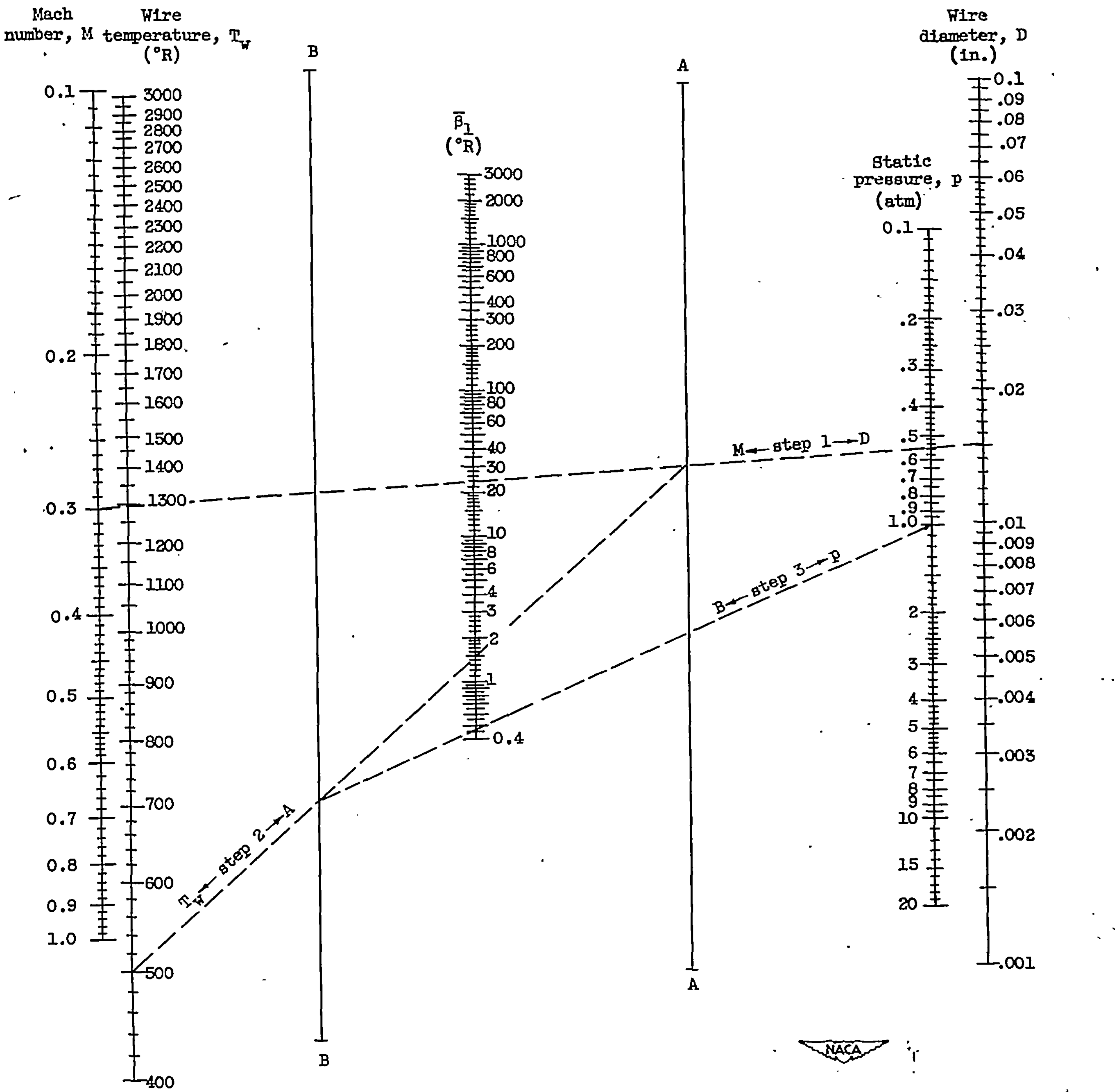


Figure 21. - Temperature distribution along wire with intermediate supports as shown in figure 2(c). One-half symmetrical distribution is shown. L, 0.5 inch; L', 0.1 inch; $\eta_p(L-L')$, 1.4.



Given: T_w = 500° R
p = 1 atm
M = 0.3
D = 0.015 in.

Procedure: Connect M to D and find intersection A.
Connect A to T_w and find intersection B.
Connect B to p and find answer $\bar{\beta}_1$.

Answer: $\bar{\beta}_1$ = 0.46° R

(a) Nomograph for computing $\bar{\beta}_1$.

Figure 16. - Nomographs for determining radiation error.

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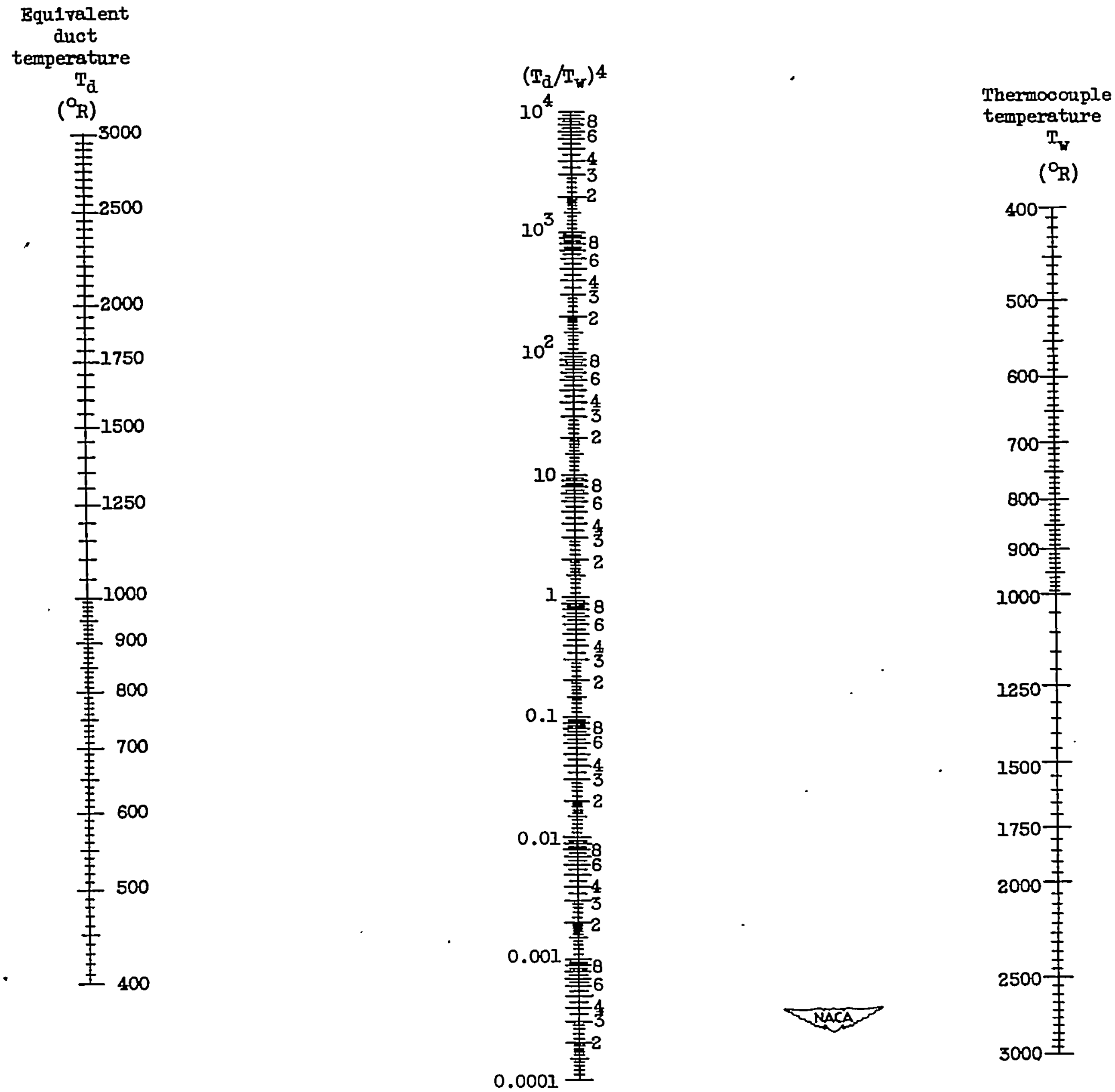


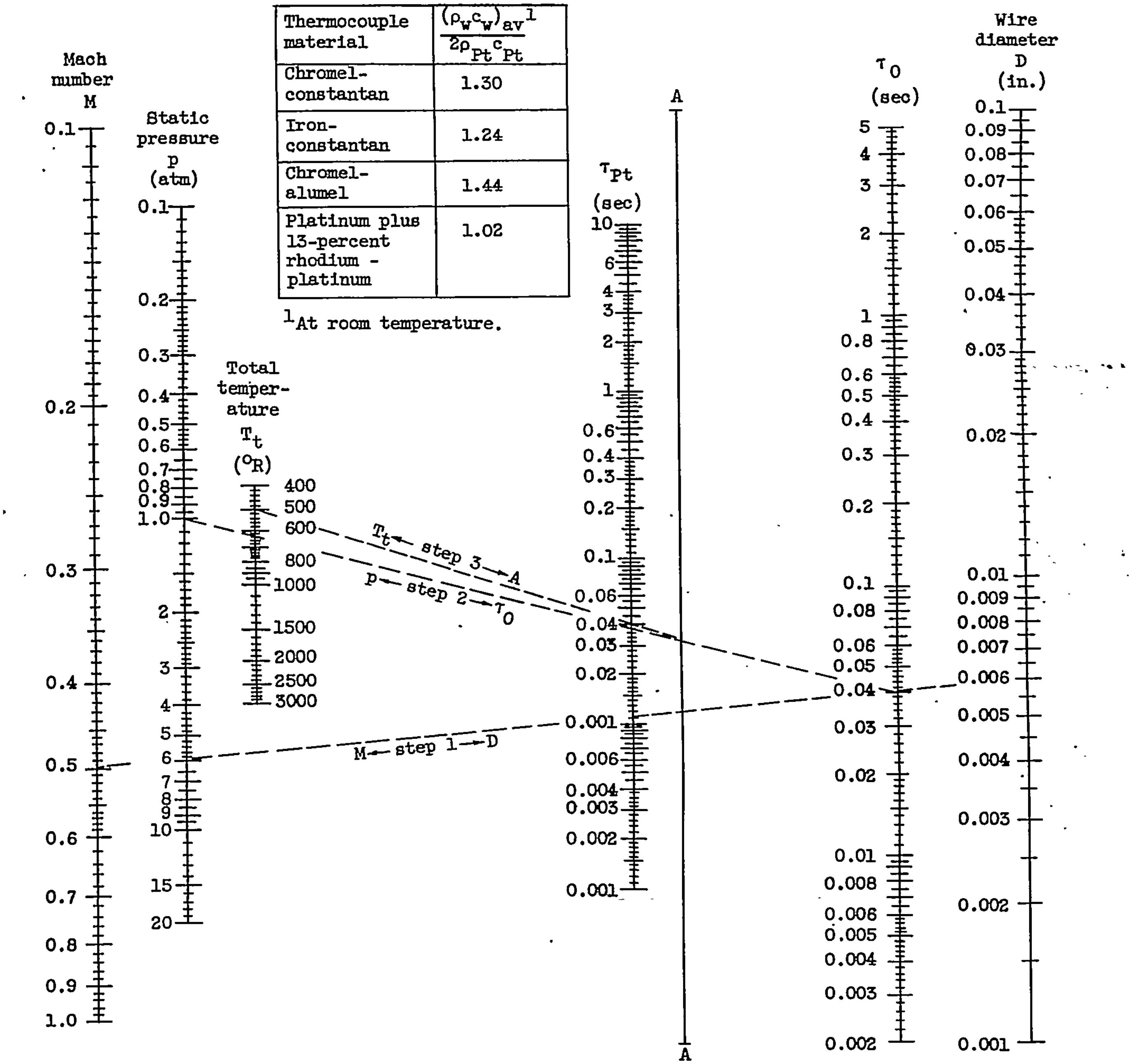
(b) Nomograph for computing $(T_d/T_w)^4$.

Figure 16. - Concluded. Nomographs for determining radiation error.

NACA-Langley - 3-18-53 - 100

Thermocouple material	$\frac{(\rho_w c_w)_{av}}{2\rho_{Pt} c_{Pt}}$ [1]
Chromel-constantan	1.30
Iron-constantan	1.24
Chromel-alumel	1.44
Platinum plus 13-percent rhodium - platinum	1.02

[1]At room temperature.



Given: M = 0.5
D = 0.006 in.
p = 1 atm
T_t = 500° R
Chromel-alumel

Procedure: Connect M to D and find intersection τ_0.
Connect τ_0 to p and find intersection A.
Connect A to T_t and find intersection τ_{Pt}.
Multiply τ_{Pt} by number in table to find τ_1.

Answer: τ_{Pt} = 0.04 sec

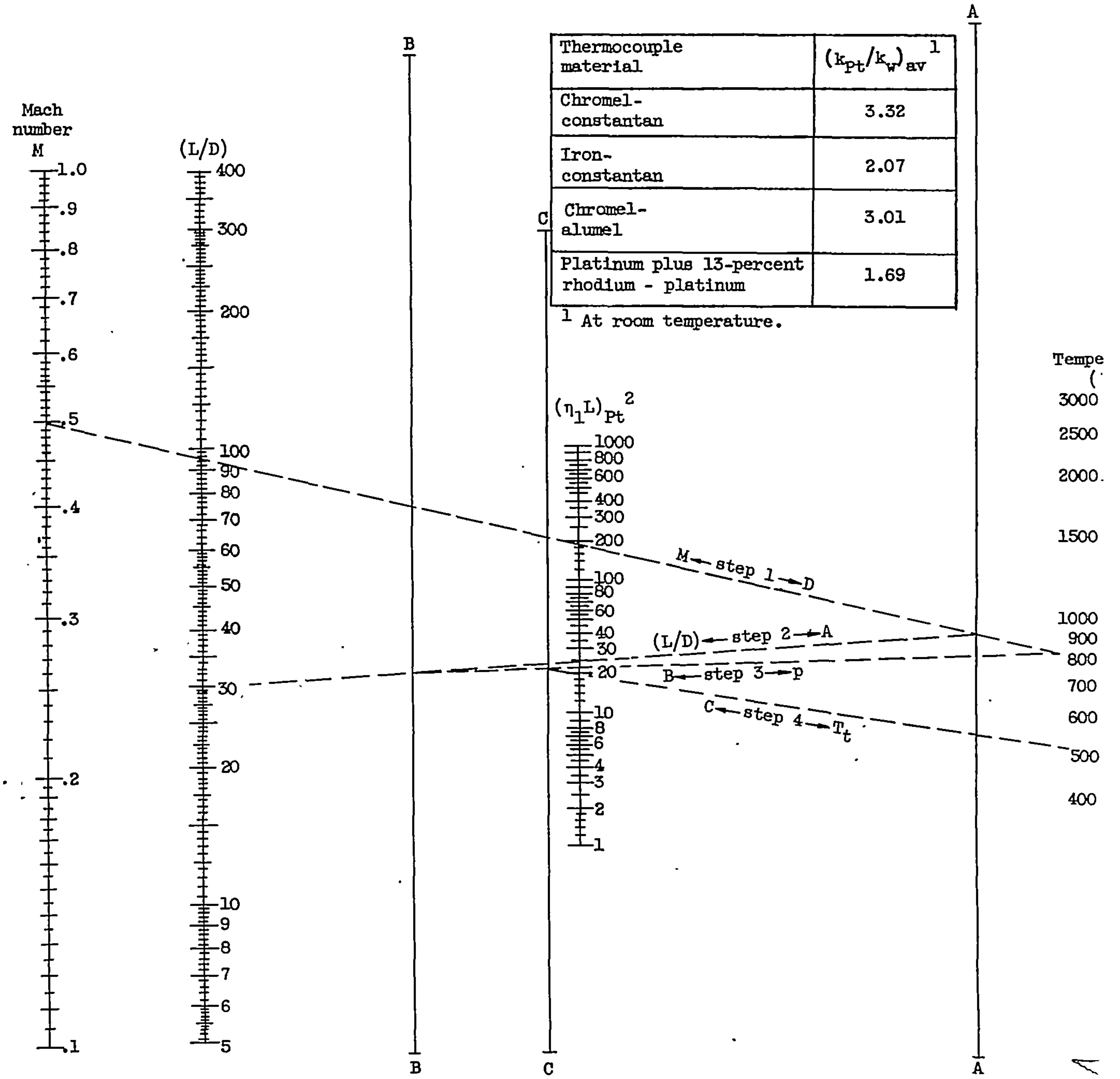
$$\tau_1 = \tau_{Pt}\left[\frac{(\rho_w c_w)_{av}}{2\rho_{Pt} c_{Pt}}\right]$$

τ_1 = 0.057 sec

Figure 17. - Nomograph for computing τ_1.

NACA-Langley - 3-18-53 - 100

2382



Thermocouple material	$(k_{Pt}/k_w)_{av}$ [1]
Chromel-constantan	3.32
Iron-constantan	2.07
Chromel-alumel	3.01
Platinum plus 13-percent rhodium - platinum	1.69

[1] At room temperature.

Given: $M = 0.5$
$D = 0.005$ in.
$L/D = 30$
$p = 1$ atm
$T_t = 500°$ R
Chromel-alumel

Procedure: Connect M to D and find intersection A.
Connect A to L/D and find intersection B.
Connect B to p and find intersection C.
Connect C to T_t and find intersection $(\eta_1 L)_{Pt}^2$.

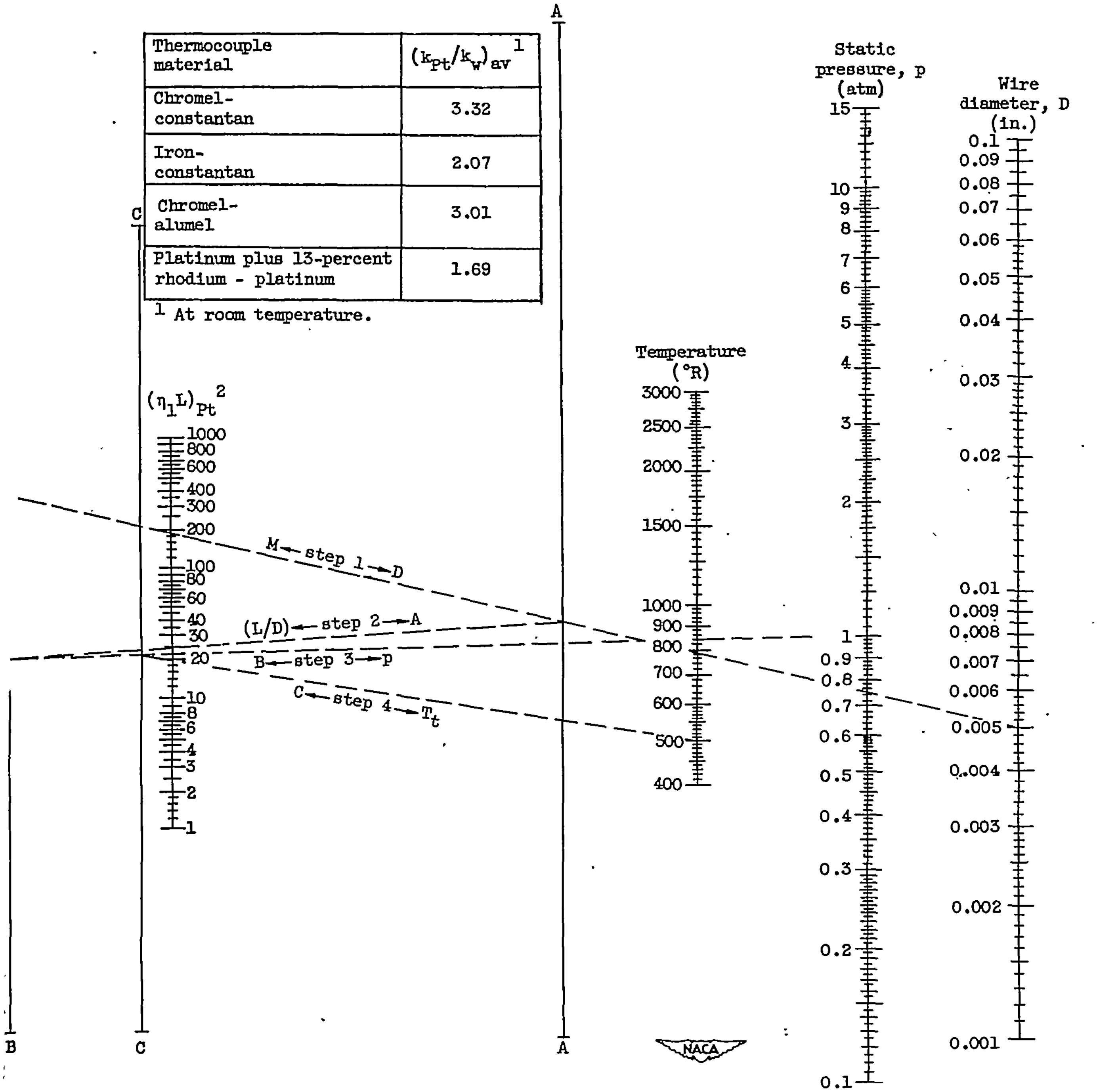
Multiply $(\eta_1 L)_{Pt}^2$ by value in table to find $(\eta_1 L)^2$.

Figure 18. - Nomograph for computing $(\eta_1 L)^2$.

NACA-Langley - 3-18-53 - 100

Thermocouple material	$(k_{Pt}/k_w)_{av}$ [1]
Chromel-constantan	3.32
Iron-constantan	2.07
Chromel-alumel	3.01
Platinum plus 13-percent rhodium - platinum	1.69

[1] At room temperature.



Procedure: Connect M to D and find intersection A.
Connect A to L/D and find intersection B.
Connect B to p and find intersection C.
Connect C to T_t and find intersection $(\eta_1 L)_{Pt}^2$.

Multiply $(\eta_1 L)_{Pt}^2$ by value in table to find $(\eta_1 L)^2$.

Answer: $(\eta_1 L)_{Pt}^2 = 20$

$(\eta_1 L)^2 = (\eta_1 L)_{Pt}^2 \left(\frac{k_{Pt}}{k_w}\right)$

$(\eta_1 L)^2 = 60$

Figure 18. - Nomograph for computing $(\eta_1 L)^2$.